Econometrics 1 (ECON 4003)
Suggested Solutions - Tutorial 2

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Abstract

This guide is supposed to be complementary to the official solutions supplied by the lecturer. All errors are my own.

Question 1

This question is first and foremost an exercise in thinking about distributions and probabilities. These are the basic building blocks of econometrics (and statistics) so it's good to get a handle on them, so you don't get lost further down the road.

a)

The table shows you the *joint distribution* of the level of advertising chosen by the two companies. Each entry tells you the probability of the event that company B chooses a certain level of advertising **and** company C chooses a certain level of advertising. In probability this is denoted as $\Pr(B = b, C = c) = f(c, b)$ where b and c refer to the possible values $\{0, 1, 2\}$ that can be chosen by the respective companies. What the question is asking you to do is to drive the *marginal* (or *unconditional*) distribution of C's advertising. In other words, we are looking for the probabilities associated with C's advertising choices **irrespective** of what B chooses to do: $\Pr(C = c) = f_c(c)$. You can do this effectively by summing across each row; i.e. $\Pr(C = c) = \Pr(B = 1, C = c) + \Pr(B = 2, C = c) + \Pr(B = 3, C = c)$.

b)

Calculating the expected value of C is quite easy once you have the marginal probability distribution (see official solutions). However I would like you to pause for a moment and think about whether this calculation actually makes sense in this case. $C = \{0, 1, 2\}$ here refers to 3 possible *ordinal* outcomes.[1] Ordinal variables are often complicated since the properties of the numbers, seldom carry through to the objects they represent. For example, in what sense is a newspaper coupon and a store display twice as good as only a coupon? I am not saying that we have to always reject analyses involving ordinal variables, but we should be careful how we interpret these statistics. For example in this case we could say that we expect to see on average 1.3 *different types* of advertising from C.

c)

You should remember the variance formula from previous studies: $Var(C) = \sum_{n=1}^{N} f(c_n)(c_n - \bar{c})^2$. This formula is usually a little cumbersome to use under exam conditions, so pay attention to the trick used in the solution to this question. It might help you in an exam situation especially if you have already calculated the expected value. Be careful to not accidentally square the marginal probability when calculating $E(C^2)$.

d)

Statistical independence is an interesting and important concept that can be summarized as: "the realization of one random variable does not affect the probability distribution of the other." Classic examples are consecutive rolls of a dice or flipping a coin. Getting heads last time does not affect your chances of flipping tails next time - the coin tosses are *independent.* More technically speaking, two events are independent, **if** their *"joint probability distribution is equal to the product of their marginal distributions."*[2]

If you would like to see why this is true, just follow along:

Proof. Assuming that: $Pr(A = a, B = b) = \Pr(A = a) * \Pr(B = b)$

Divide by LHS: $\frac{Pr(A=a,B=b)}{\Pr(B=b)} = \Pr(A = a)$

Use Bayes Law: $\Pr(A = a) = \frac{Pr(A=a,B=b)}{\Pr(B=b)} = \Pr(A = a|B = b)$

Obviously the same is true for $\Pr(B = b) = \Pr(B = b|A = a)$

Q.E.D. ■

What this is telling you is that $\Pr(A = a)$ is not affected by your knowledge that the event $B = b$ occured, it doesn't provide you with any additional information. The exercise just asks you to find a simple counterexample. As for exam

[1] Ordinal variables should not be confused with categorical variables. Ordinal variables have a certain rank ordering to them, even though the exact distance between the outcomes might not be measurable, whilst no ordering is apparent between categorical outcomes. For example $\{Coke, Pepsi, Sprite\}$ are categorical outcomes whilst $\{good, okay, bad\}$ are ordinal ones.

[2] Important information: I might have told some of you that having the product of the marginal probabilities be the joint does not guarantee statistical independence. This is incorrect, my apologies - if the joint is the product of the marginals then they are independent by definition. What I was thinking about was covariance: i.e. if two random variables are independent, then they have 0 covariance. However, 0 covariance does not guarantee independence.

tips this means that you will have to check every single possbile combination until you find a counterexample.

e)

Here we have to find the marginal probability distribution of B - $\Pr(B = b)$. Now instead of summing across rows we simply sum up the columns of the table.[3] After you found this it's straightforward to write down the cost outlay associated with each level of advertising. This is so easy since A is merely a linear transformation of B.

f)

I think it's quite helpful for this exercise to have a collection of quick and easy rules for expectations and variances so I will just list some that will be helpful for following the calculations in the solutions. For this suppose that X and Y are random variables and that a, b, c are constants.

$E(aX + bY + c) = aE(X) + bE(Y) + c$ i.e. the expectation of a constant is always a constant (makes sense?) and the expectation of a sum is the sum of the expectations.

However, generally $E(XY) \neq E(X)E(Y)$! This equality holds whenever X and Y are independent, but not in general.

$Var(X + a) = Var(X)$ adding a constant does not change the variance, it just shifts the position of the distribution, not the spread.

$Var(aX) = a^2 Var(X)$ don't forget to take the square of the scaling constant!

$Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$ - doesn't this look suspiciously like the expansion of $(a + b)^2$?

With these in mind you should be able to follow the calculations. First use the formula just above to derive an expression for $Cov(A, B)$. Second use the definition of advertising expenditure to substitute A. The rest is just some tedious algebra.

Question 2

a)

Just remember the little rules above: the expectation of a sum of two random variables is the sum of the expectations. i.e. if you are expecting to get £50 from your grandmother and £100 from your grandad for Christmas, you should expect to get £150 in total.

[3] Pro-tip: We know that probabilities always need to sum to 1, so once you arrive at the last option it can often be easier to calculate the N'th Probability by: $Pr(B = N) = 1 - \sum_{n=1}^{N-1} Pr(B = n)$.

b)

Same strategy here $Cov(X, Y) = 0$ due to statistical independence and don't forget that $\frac{1}{2}$ here is a constant so you have to square it when you take it outside of the variance operator. Also, notice what the result suggests: the variance of the mean of two independent variables is smaller than the variance of each individual variable. This is an important result that can be further generalized to what is called the *Law of large Numbers*. To illustrate this, think about the variance of the **average** for a large number of independent outcomes.[4]

$$\lim_{N\to\infty} Var\left(\frac{1}{N}\sum_{n=1}^{N} X_n\right) = \lim_{N\to\infty} \frac{1}{N^2}\sum_{n=1}^{N} Var(X_n) = \lim_{N\to\infty} \frac{\sigma^2}{N^2} = 0$$

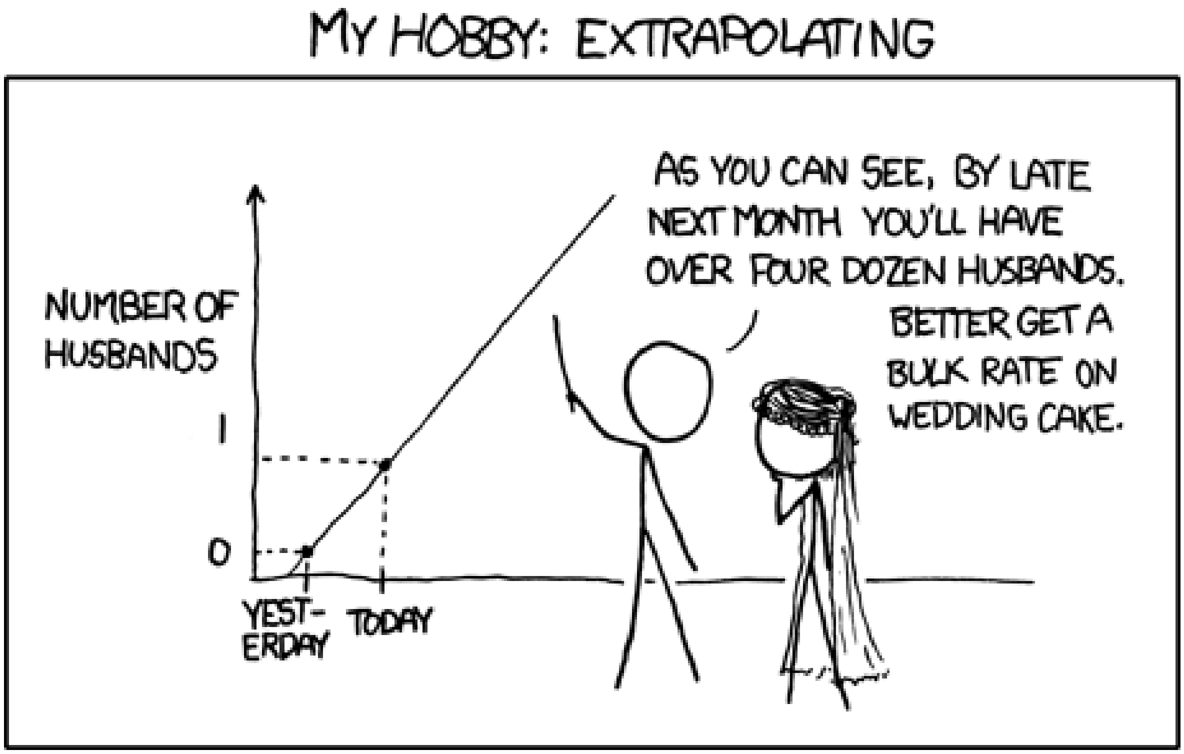

Figure 1: Source: https://xkcd.com

[4]Again, this is not the variance of an individual outcome, but the variance of the measured average over many outcomes.