Econometrics 1 (ECON 4003)
Suggested Solutions - Tutorial 6

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Abstract

This guide is supposed to be complementary to the official solutions supplied by the lecturer. All errors are my own.

Question 1

We have seen before, that including an intercept will set the mean of the residuals to 0. You can see this somewhat intuitively by using the definition of the intercept for any given value of :

$$\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X} \tag{1}$$

simply replace $\bar{Y}$ with the following expression:

$$\bar{Y} = \hat{\beta}_1 \bar{X} + \bar{u} \tag{2}$$

since the mean observation is simply the mean predicted value plus the mean error. Hence:

$$\hat{\beta}_0 = \hat{\beta}_1 \bar{X} + \bar{u} - \hat{\beta}_1 \bar{X} \tag{3}$$

$$\hat{\beta}_0 = \bar{u} \tag{4}$$

so the intercept takes on the mean residual value, that would pervail in a model where there is no intercept. Neat, don't you think?

Question 2)

a)

The difficult step to get started here is a concepetual one: What we are trying to assess here is what kind of bias is introduced into our estimate of $\tilde{\beta}_1$ by forcing the intercept to be 0. For this we are assuming that the population regression modle is true, i.e. $Y = \beta_0 + \beta_1 X + u$. Once you got this step it's simply just a bunch of algebra to find the desired expression.

b)

It is almost too obvious that when the true intercept is equal to zero, then we do not hurt ourselves by imposing that assumption. A little more interesting is the suggestion that if the explanatory variable is mean 0, this is also the case. This suggests some interesting applications for demeaned data.

c)

If you want to see how to derive the conditional variance for $\tilde{\beta}_1$ you can go to the appendix of lecture 3 (slide 74 f.) and simply replace our estimator. OR you could simply imagine that you were dealing with a situation in which $\beta_0 = 0$ was true. In that case you can simply "see" what the conditional variance would be for a case with $\bar{X} = 0$.

d)

In a way by "anchoring" our regression line at the origin we have taken away some of it's flexibility. This means it can't move around too drastically and hence has a lower variance. What is important to note is that it has smaller variation around the **biased** estimate!!!

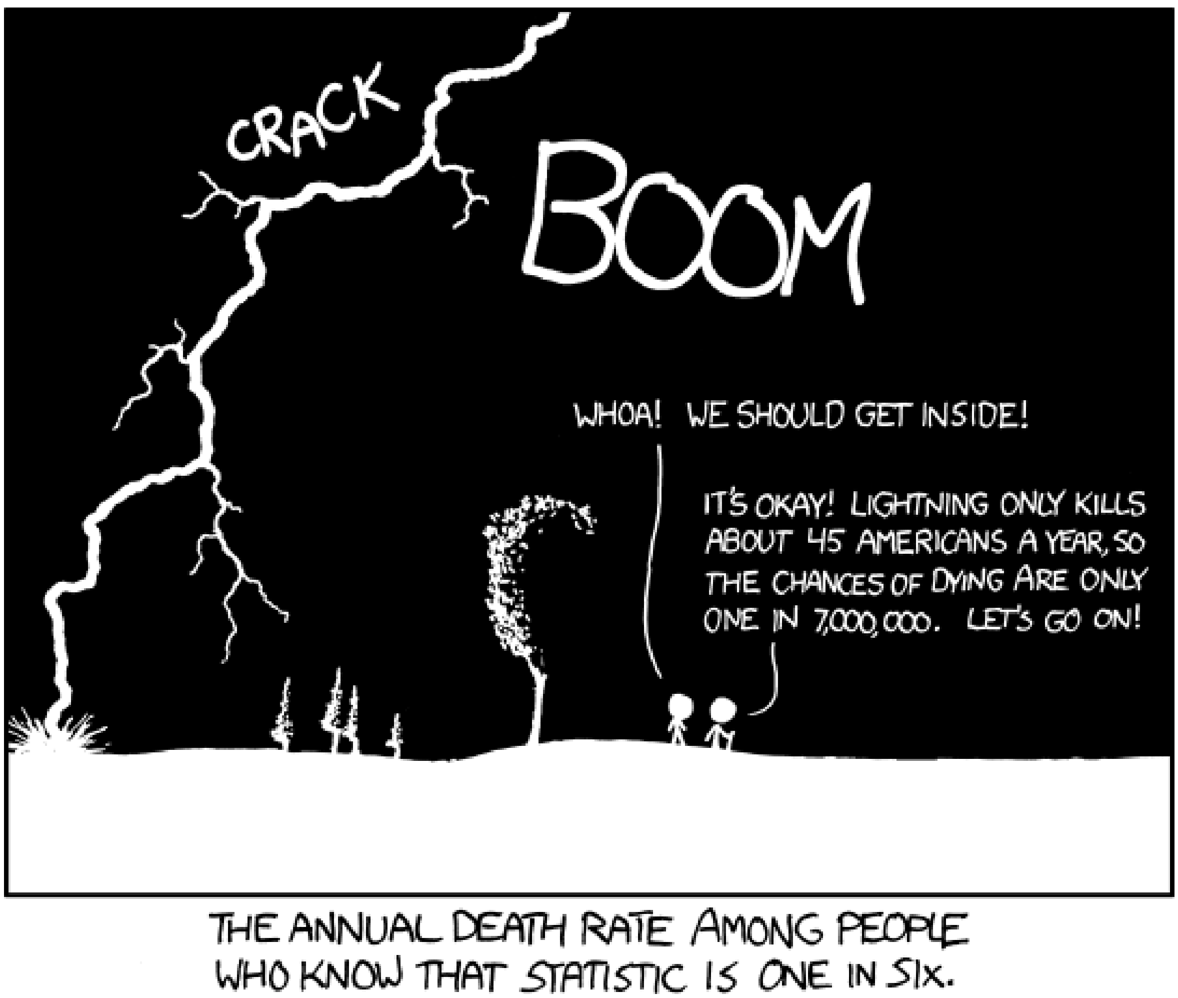

Figure 1: Source:https://xkcd.com