Economics 1B
Suggested Solutions - Seminar 1

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Abstract

This guide is supposed to be complementary to the official solutions supplied by the lecturer. All errors are my own.

Exercise 3.1

Q1

a) $\frac{7}{20}$ b) $\frac{22}{25}$ c) $\frac{5}{2}$ d) $\frac{7}{40}$ e) $\frac{1}{500}$

Q2

Note that $x\%$ can be expressed as $\frac{x}{100}$.

a) 1.2 b) 7.04 c) 2190.24 d) 62.72

Q11

1) 1985

2&3)

	1985-1990	1990-1995	1995-2000	2000-2005
Public	30%	52.3%	13.1%	9.4%
Private	25%	52%	10.6%	11.1%

4) Generally public transport costs increase at a faster rate than private costs.

Exercise 3.2

Q6

Use the compound interest formula:

$$FV = PV(1 + \frac{r}{n})^{n*t} \tag{1}$$

as n grows very large, we approach the continuous case:

$$\lim_{n->\infty} FV = PV(1+\frac{r}{n})^{n*t} = PV * e^{r*t} \tag{2}$$

a) 13948 b) 14157 c) 14342 d) 14381

Exercise 3.3

Q2

a) This question is related to the previous one, but we have to account for the extra deposit that is made at the begining of each year. For example, the value of the first deposit after 10 years is:

$$FV = PV(1+\frac{r}{n})^{10*n} \tag{3}$$

the value of the second deposit is going to be

$$FV = PV(1+\frac{r}{n})^{9*n} \tag{4}$$

as the interest had one less year to accrue.
Overall the value of all 10 deposits is going to be:

$$FV = \sum_{t=1}^{10} PV(1+\frac{r}{n})^{t*n}$$

Now take $n = 1$

$$FV = \sum_{t=1}^{10} PV(1+r)^{t}$$

This looks almost like a geometric series, To remind you, in a geometric sum we have the following formula:

$$a + ar + ar^2 + ... + ar^{n-1} = \sum_{t=o}^{n-1} ar^t = a\left(\frac{1-r^n}{1-r}\right)$$

The only problem is, that the geometric sum formula ranges from $t = 0$ to $t = n-1$. To account for this we will multiply and divide by $(1+r)$, effectively shifting all the values of t up by one. Hence we have:

$$\begin{aligned} FV &= \frac{(1+r)}{(1+r)} \sum_{t=1}^{10} PV(1+r)^t \\ FV &= (1+r) \sum_{t=0}^{9} PV(1+r)^t \\ FV &= PV\left(\frac{1-(1+r)^9}{1-(1+r)}\right)(1+r) \end{aligned}$$

Plugging in the numbers gives us the result: 78227£

b) To calculate the semi-annual interest we can use a little trick, namely: $a^{b*c} = a^{b^c}$ to replace $(1+\frac{r}{n})^{t*n}$ with $\left((1+\frac{r}{n})^n\right)^t$. With this "new" interest rate[1] we can do use the same formula as above to get: 78941£.

Exercise 3.4

Q1

All we have to do is to rewrite the future value formula to find the present value expression:

$$PV = \frac{FV}{(1+\frac{r}{n})^{t*n}}$$

so a) 5974£ and

$$PV = \frac{FV}{e^{t*r}}$$

b) 5965£.

Q5

Actors in economics are often supposed to be indifferent with respect to time periods. Money now, or money tomorrow, doesn't matter - what is relevant is the total value. Usually this means we prefer money today, because if we get it now we can put it in the bank and earn interest which means we will have more in the future.

Net Present Value tries to capture tradeoffs, by comparing the value of some payment in the future, today (i.e. the discounted PV from the question above) with the cost today. For example, if you can trade in one pound today for one pound tomorrow, you are effectively loosing value, as you could have put the money in a bank account and earned interest on it. So NPV is the present value of some future income stream $\frac{FV}{(1+\frac{r}{n})^{t*n}}$ **net** of the present costs.

Project 1 NPV = $\frac{15000}{1.09^4} - 10000 = 626$
Project 2 NPV = $\frac{25000}{1.09^5} - 15000 = 1248$

[1] This is sometimes called the "effective interest rate".

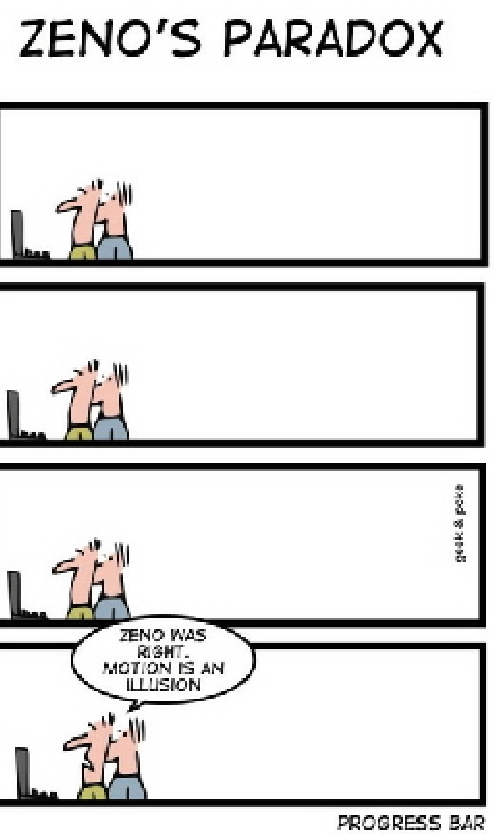

Figure 1: https://www.datamation.com/news/tech-comics-zenos-paradox-1.html