Economics 2B
Suggested Solutions - Tutorial 1

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Abstract

This guide is supposed to be complementary to the official solutions supplied by the lecturer. All errors are my own.

Question 1

This question is a good start to get your head around some basic concepts and notations. A lot of this course is just rewriting equations, so we should probably start by deriving the main equation here. Start with the set of equations describing all the relevant Quantities:

$$Y = C + I + G \tag{1}$$
$$C = c_0 + c_1(Y - T) \tag{2}$$
$$T = T \tag{3}$$
$$G = G \tag{4}$$
$$I = I \tag{5}$$

Note that the last 3 equations are somewhat superfluous, but I added them for completeness. The aim of much of economic theory is to reduce economic relationships to their *fundamental* parts. It is not easy to precisely say what is meant by *fundamental*, but for the time being you can think of a variable as being fundamental if it does not depend on other variables of the system. So for example here c_0 is fundamental, but C is not. Another term for fundamental is **exogenous,** while non-fundamental variables are called **endogenous**. Here, I, G, T, c_0, c_1 are **exogenous** while $Y\&C$ are **endogenous;** i.e. we can rewrite the latter as functions of the former.

Now that we have these notions pinned down, we just have to substitute out all endogenous variables with our exogenous ones. To do this simply insert the function for C into the first equation and do a little algebra:

$$
\begin{aligned}
Y &= c_0 + c_1(Y - T) + I + G \\
Y - c_1 Y &= c_0 - c_1 T + I + G \\
(1 - c_1)Y &= c_0 - c_1 T + I + G \\
Y &= \frac{1}{(1 - c_1)} (c_0 - c_1 T + I + G)
\end{aligned}
$$

a & b)

Now we are ready to work on the questions. There are two ways to approach these questions: The first way is to define two values of G (and T): G^1 and $G^2 = G^1 + 1$, plug in G^1 and G^2 into the equation for Y and calculate the difference between the two outcomes.

$$
\begin{aligned}
Y^1 &= \frac{1}{(1 - c_1)} \left(c_0 - c_1 T + I + G^1\right) \\
Y^2 &= \frac{1}{(1 - c_1)} \left(c_0 - c_1 T + I + G^2\right) \\
Y^2 - Y^1 &= \frac{1}{(1 - c_1)}
\end{aligned}
$$

However, it is a lot easier, if you realise that is asked here is the *derivative of Y wrt. to G and T:* $\frac{\partial Y}{\partial G}$ and $\frac{\partial Y}{\partial T}$.[1] And since the equation has a nice linear form it is really easy to find the derivatives:

$$
\begin{aligned}
\frac{\partial Y}{\partial G} &= \frac{1}{(1 - c_1)} \\
\frac{\partial Y}{\partial T} &= \frac{-c_1}{(1 - c_1)}
\end{aligned}
$$

c)

In order to answer this question it is helpful to look at the equations again, especially at the one for Y and C. When G goes up, Y increases 1 to 1. However when T goes up, C only falls by $c_1 < 1$. In other words, some of the increased tax bill is financed by reduced savings, which is why the impact on output is smaller. Conversely, if taxes are reduced some of that windfall is saved and not recirculated into the economy.

d)

Lets use a shortcut for this: insert $T = G$ and take derivatives:

$$
Y = \frac{1}{(1 - c_1)} (c_0 - c_1 G + I + G)
$$

[1] As a rule of thumb for economics you can remember that *everything* has to do with derivatives.

$$\frac{\partial Y}{\partial G} = \frac{(1-c_1)}{(1-c_1)} = 1$$

so the multiplyer is 1. If you increase government spending by one unit and finance that by taxing one extra unit output overall increases by 1 unit. Amazing, right?

e)

There are two answers to this: The first is mathematical - c_1 simply drops out of the derivative, so it clearly doesn't influence the multiplier. The second is economic - the marginal propensity to consume affects both parts of the multiplier, albeit in opposite directions. A higher value of c_1 means that more of the extra income from government spending is recirculated, but similarly it means that more of the tax increase must be financed by reduced expenditure rather than by reducing saving. On balance the two effects cancel each other out.

Question 2

a)

Here we have got a new endogenous variable T and two new exogenous variables t_0 and t_1.

$$Y = C + I + G \tag{6}$$
$$C = c_0 + c_1(Y - T) \tag{7}$$
$$T = t_0 + t_1 Y \tag{8}$$
$$G = G \tag{9}$$
$$I = I \tag{10}$$

We basically follow the same steps as before:

$$Y = c_0 + c_1(Y - (t_0 + t_1 Y)) + I + G$$
$$Y - c_1 Y + c_1 t_1 Y = c_0 - c_1 t_0 + I + G$$
$$Y = \frac{1}{1 - c_1 + c_1 t_1}(c_0 - c_1 t_0 + I + G)$$

b)

The multiplier here refers to the "autonomous spending multiplier" i.e. $\frac{\partial Y}{\partial c_0}$:

$$\frac{\partial Y}{\partial c_0} = \frac{1}{1 - c_1 + c_1 t_1}$$

it is easy to see that the size of the multiplier gets smaller the bigger t_1 is. This also makes economic sense: if taxes increase with economic activity, then people have less money to spend.

c)

Fiscal policy in this setting is automatic in the sense that the government doesn't have to actively change taxes as they are partly determined by income. Remember that T refers to the total/gross tax take, just like Y referrs to total/gross output. This mechanism is stabilizing since the smaller multiplier means that output responds less to changes in exogenous parts of income, such as c_0 or I.

Question 3

a)

This is exactly the same problem as in 2a.

$$Y = \frac{1}{1 - c_1 + c_1 t_1} (c_0 - c_1 t_0 + I + G)$$

b)

Taxes here refers to the total tax intake T. Don't confuse it with the tax rate t_1. From our tax equation we know that $T = t_0 + t_1 Y$ and luckily we have just calculated output, hence:

$$T = t_0 + t_1 \left(\frac{1}{1 - c_1 + c_1 t_1} (c_0 - c_1 t_0 + I + G) \right)$$

c)

Again, the easiest way to answer this question is to take derivatives first, and use the mathematical results to aid your economic intuition. So we have

$$\frac{\partial Y}{\partial c_0} = \frac{1}{1 - c_1 + c_1 t_1}$$

and

$$\frac{\partial T}{\partial c_0} = \frac{t_1}{1 - c_1 + c_1 t_1}$$

.

Note how the effect on taxes is smaller than on output. Make sure you understand why.

In this example it is probably very easy to tell a consistent story of what happens, but it is good practise to get in the habit of analysing these relationships mathematically. As models become more complex, it gets harder and harder to grasp what is going on and our intuitions are easily lead astray.

d)

Lets see if we can derive the balanced budget multiplier. For simplicity, lets assume that $t_0 = I = 0$.

So we have:

$$Y = C + G \tag{11}$$
$$C = c_0 + c_1(Y - T) \tag{12}$$
$$T = t_1 Y \tag{13}$$
$$G = T \tag{14}$$

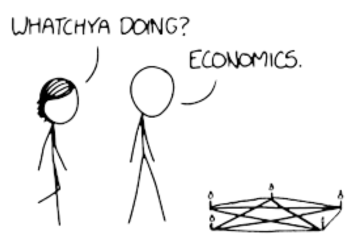
Start with output and substitute in the other equations:

$$Y = C + G$$
$$Y = c_0 + c_1(Y - T) + T$$
$$Y = c_0 + c_1(Y - t_1 Y) + t_1 Y$$
$$Y = c_0 + Y(c_1 - c_1 t_1 + t_1)$$
$$Y(1 - (c_1 - c_1 t_1 + t_1)) = c_0$$
$$Y = \frac{c_0}{1 - c_1 + c_1 t_1 - t_1}$$

Finally we need to take derivatives to get the multiplier:

$$\frac{\partial Y}{\partial c_0} = \frac{1}{1 - c_1 + c_1 t_1 - t_1}$$

which is bigger than the multiplier without a balanced budget.



Source: https://xkcd.com/