Cognitive Skill Biased Technological Change, Income & Wealth Inequality in the UK*

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Abstract

This chapter proposes a model to study the effect of differentiated, cognitive skill biased, technological change on income and wealth inequality. It is one of the first attempts to combine elements of the "Task-Skill" literature with a heterogenous agent incomplete market model. The model includes a structural production environment that accounts for differentiated skill demand on the firm side and multidimensional skill supply on the workers side. Using measures of cognitive and non-cognitive skills from a comprehensive panel dataset for the UK (Understanding Society), I calibrate the model with appropriate micro-estimates. The calibrated model manages to capture many of the features of the income process observed in the data and provides additional features beyond other, more commonly used approximation techniques. I then use the model to assess the impact of cognitive task biased technological (CBTC) change due to increased Computer usage in the UK over the period 1980 - 2016. The model suggests that CBTC can account for the bulk of increases in labour income inequality observed over that period, and is generally consistent with stylized facts about changes to wealth inequality.

Keywords: Between- & Within Group Inequality, Technological Change, Cognitive Skills

JEL codes: D31, J24, O11

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1 Introduction

In recent years, economists and laypeople alike have become increasingly focussed on income and wealth inequality as the defining economic and political issues of our times.[1] In Britain, for example, income and wealth inequality seem to have increased for the last decades (c.f. Hills et al. (2013), Roine & Waldenstroem (2015)). The general perception is that the two phenomena do not occur independently, but rather feed of one another: those with high incomes are able to stash away their earnings in lucrative financial investments or finance increasingly expensive education for their offspring, leading to increasing concerns about the implications for intragenerational mobility, equality of opportunity and distributional justice. Yet despite the connection between the two issues, the complexity of each phenomenon has so far mostly precluded a unified analysis.

The notion that precautionary savings are a key driver of wealth inequality is a key insight of the incomplete markets models of the Bewley-Huggett-Aiyagari variety (c.f. Aiyagari (1994), Bewley (1986), Huggett (1993), Imrohoroglu (1989)). In these models uncertainty about future income induces individuals to accumulate precautionary savings, leading to an endogenous distribution of wealth.

In order to remain theoretically simple and computationally feasible, these models tend to take a certain degree of income inequality as given,[2] to produce a given level of wealth inequality endogenously. Over the last decades these models have become the benchmark for studying wealth inequality and found numerous applications to different macroeconomic problems (see Heathcote et al. (2009), or Quadrini & Rios-Rull (2015) for a survey).

On the other side of the literature, economists have worked for the last decades to generate an understanding of the nature of income inequality, that focusses on the importance of technological change and the changing supply of skills,[3] most aptly summarized by Jan Tinbergens (1974, 1975) *"Race between Education and Technology"* (c.f. Acemoglu & Autor (2011), Katz & Murphy (1992), Goldin & Katz (2007)). Their insights suggest, that incomes are affected by a complex interplay of the technology driven demand for and the supply of differentiated skills.

The connection between these two strands of literature should be of interest to econo-

[1] See for example the huge range of popular books that deal with the subject matter, such as: Atkinson (2015), Milanovic (2016), Piketty (2013), Stiglitz (2012), and many more.

[2] Or more precicely, a given distribution of individual income realizations.

[3] There are of course other prominent explanatory frameworks, such as the majorily trade based Heckscher-Ohlin model, or models based on political economy chanels (see Vivarelli (2014) for an overview).

mists studying either phenomenon: If technology generates income inequality, and income inequality is the main ingredient in the generation of wealth inequality, then the case could be made that both issues have the same root cause.

Technological change is not without a direction - when Frey & Osborne (2013) published their report claiming that 50% of jobs were at risk of being automated within a generation, they attached fairly precise estimates to the automatization probabilities of different occupational groups. Claims such as these are based on the insights of Autor et al. (2003) and many others, that some work activities lend themselves more readily to being automated than others. So far the literature has mainly identified so called *"routine"* tasks - repetitive, primarily manual work activities - as victims of automatization.[4] This *Routinization Hypothesis*, has somewhat illuminated the long observed trend of *"Labour Market Polarization"* (c.f. Goos et al. (2009), (2014), Goos & Manning (2007)), that describes an increase in the demand for labour in the tail ends of the wage distribution.

Apart from substituting for routine tasks, computer technology also plays a role in complementing complex human activities, increasing the return to analytic skills and increasing the incomes of highly educated workers.

It is clear from the outset, that technological change has wide ranging effects on the distribution of incomes in a society - effects that can hardly be captured by any single distributional parameter. This suggests, that it might be useful to replace log-normal income processes with more realistic, structural processes, which better capture the complex empirical reality of income risk (c.f. Bayer & Kuhn (2018), Guvenen et al. (2015), Hubmer (2018)). Once adequately described, the changes in the distribution of income, due to technological change can be used as inputs to understanding changes to the distribution of wealth.

The aim of this chapter is to build a model that captures the spirit of this idea and investigate the question to what extend the rise in income and wealth inequality can be attributed to the advent of advanced Information and Communication Technologies (ICT) in the 1980's. To this end, I insert a technology driven model of income determination, that is based on the *"Skill-Weights-Approach"* (Laezar (2009)) into a standard Aiyagari (1994) model.

In this case income depends on the skill-set and occupation match of a worker while

[4] In principle, however any work task is capable of being automated, given suitable technology and economic incentives, as aptly demonstrated by Acemoglu & Restrepo (2015).

the returns to skills depend on the supply-demand relationship in general equilibrium. For simplicity the constituent parts of the income process remain purely exogenous, as in a standard HIM application, but by taking a step back and modeling the relationship between the demand and supply of different skills explicitly, the model can shed some light on the potential effects of differentiated technological change. The aim in this is not to create a model that perfectly captures the determinants of income, but to add just enough complexity so that the model can capture the theoretical entities, and still be simple enough to be calibrated with available data and solved easily with known techniques.

After calibrating a stationary version of the model, using a sample of workers from the UK's *Understanding Society* dataset, I use the model to explore the response of the endogenous variables to heterogenous technological change, as computers penetrate different occupational niches at varying rates. In this the model is uniquely suited for the study of income and wealth inequality dynamics over the short to medium term, as it allows me to trace the effects of technological change on both variables simultaneously.

The results suggest that increasing adoption of computers and computerized equipment over the period $1980 - 2016$ can account for around 90% of mean wage growth and 80% of the growth of income inequality. Furthermore the models predictions are in line with stylized facts about the evolution of wealth inequality. This suggests, that cognitive skill biased technological change might be a good approach towards explaining the general trend of increasing income & wealth inequality for a large sample of the working population in the UK.

1.1 Related Literature

The chapter relates to roughly four different strands of literature:

Firstly, it relates to the literature on technological change in heterogenous agent models: starting with the seminal work of Krussell & Smith (1998), a large literature has developed around the notion of aggregate uncertainty in HIM models, which tends to be indicated by aggregate technology shocks. This chapter studies the effect of a specific type of aggregate productivity shock: *cognitive skill biased technological change*. The main methodological contribution is to provide a working example of *heterogenous* technological change. As such it differs from the rest of the literature in two important ways: 1. The aggregate technology is differentiated, and therefore technological change can have nuanced effects on different groups of workers. 2. Technological change is certain and permanent in so far that I place the emphasis on the transition path to a new equilibrium, rather than a one off shock (c.f. Rios-Rull (1999)).

Secondly, the chapter relates to the literature on Task Biased Technological Change (TBTC) (c.f. Acemoglu & Autor (2012), Acemoglu & Restrepo (2015)), in so far that it embraces the notion that technological change can have different effects depending on an individuals comparative advantage. Workers are subject to two exogenous forces that determine their idiosyncratic labour productivities: their skill-set and their occupation match. As the technological environment changes due to *cognitive skill biased technological change,* different skill-occupation states become more productive whilst others fail to keep up, leading to an endogenous increase in wage inequality. Importantly, however the model abstracts from the endogenous assignment of workers to tasks and instead focusses on the feedback effect between income inequality, precautionary savings behavior and capital accumulation.

It also contributes to the study of the effects of computerization, by showing that the increasing return to cognitive skills associated with increased computer usage is sufficient to explain the bulk of the growth in income inequality and wage growth since the 1980's, as well as some stylized facts about growing wealth inequality.

Thirdly, the chapter contributes to the literature on between- and within group inequality and labour market polarization (c.f. Angelopoulos et al. (2017), Cortes (2016), Goos & Manning (2003), Goos et al. (2014), Kambourov & Manovski (2009)), by offering a simple and attractive framework for studying these topics under different technological change scenarios. In the same vein, the chapter extends research into the relationship between changes to income uncertainty and skill premia (c.f. Heathcote et al. (2010), Guvenen & Kuruscu (2012), Slavik & Yazici (2018)), by generating meaningful between group wage premia from a simple technological process. As the model is rooted in the task-skill literature, the resulting group distinctions are occupation based, rather than the more familiar distinctions based on education, even though some similarities might be inferred. Similarly wage premia refer to occupation wage premia rather than *skill* or *degree premia.* Extending the model to include these additional distinctions might be an interesting extension.

Fourthly, the chapter contributes to the literature on Task and Skills (c.f. Autor et al. (2003), Autor (2013), Sanders & Taber (2012)) and in particular the *"Skill-Weights"* literature (c.f. Autor & Handel (2013), Laezar (2009)), by providing an alternative way of calibrating skill weights using directly observed skill measures.

The *"Task-Skill"* framework (c.f. Autor et al. (2003), Autor (2013), Sanders & Taber (2012)), suggests that productivity depends crucially on the match between a workers multidimensional skill set and their job's multidimensional skill demand. A lot of recent work in this area (c.f. Yamaguchi (2012), Guvenen et al. (2017), Lise & Postal-Viney (2017)) has already shown that this approach can be successfully employed to explain life-cycle income profiles.

Traditionally, skill weights are calibrated using task survey data, such as is provided by the US O*Net database. Here I use available measures of skills to infer the return to the corresponding tasks via econometric analysis. This approach avoids the selection problem, inherent in models of occupational choice, and provides some evidence for the validity of common survey based approaches (c.f. Autor & Handel (2013), Bisello (2013), Firpo et al. (2011), Gathman & Schoenberg (2010), Rohrbach-Schmidt & Tiemann (2013)).

The rest of the chapter is structured as follows: Section 2 presents the production environment of the economy and elaborates on the role of differentiated skill supplies & demands; Section 3 presents the households problem; Section 4 gives the definition of the recursive, stationary equilibrium; Section 5 provides a simple illustrative example; Section 6 describes the calibration of the model; Section 7 compares the properties of the model generated income process with the data and outlines the implications for wealth inequality; Section 8 provides a decomposition of income risk into cognitive and physical skill risk; Section 9 presents the dynamic effects of cognitive skill biased technological change due to increased computerization & Section 10 concludes.

2 Production Environment

2.1 Representative Firm

There is a representative, competitive firm that produces a single homogenous good (Y) using a customary Cobb-Douglas production technology using capital (K), as well as labour services (L) as inputs:[5] [6]

$$Y = K^{\alpha} L^{1-\alpha} \tag{1a}$$

$$\alpha \in (0,1) \tag{1b}$$

Labour services L are differentiated and derived from the combination of intermediate labour services l_n, supplied by different occupations, or *occupation islands,* denoted by $n = \{1, .., N\}$:

$$L = \sum_{n=1}^{N} l_n \tag{2}$$

On these islands, intermediate labour services l_n are produced by a weighted linear combination of a number of general skills (e.g. Manual, Cognitive, Interpersonal, etc.), denoted by $m = \{1, .., M\}$.[7] S_m^n summarizes the total amount of skill m supplied by workers on the occupation island n.

$$l_n = \sum_{m=1}^{M} \lambda_m^n S_m^n \tag{3a}$$

$$\infty > \lambda_m^n \geq 0, \forall m, n \tag{3b}$$

The *occupation islands* can be thought of as different departments of the representative firm, that perform different work activities, each using a specific production technology, contributing to the production of the homogenous output. The firm has no influence over the allocation of workers across these departments, but is able to purchase units of skills directly from each island and pay differentiated wages.

Each occupation n is associated with a set of weights $\lambda^n = (\lambda_1^n, ..., \lambda_M^n)$.[8] The weights λ_m^n indicate how productive the general skill m is in the production process in occupation n. It has long been argued that different work tasks require different combinations of skills, so that the same worker might be more or less productive, depending on which work activity he applies his skill-set to (c.f. Autor et al. (2003), Acemoglu & Autor (2012)).

[5] Since all aggregate quantities are constant in the stationary equilibrium, time subscripts are omitted in this part of the exposition.

[6] There is no aggregate productivity term A in the production function, since all relevant productivity is contained in the occupation-skill specific productivity parameters.

[7] In a sense these skills correspond to the types of "Task Specific Human Capital" in Gathmann & Schonberg (2010).

[8] Sometimes I will refer to $\lambda = (\lambda^1, ..., \lambda^N)$ which denotes the set of all sets of weights.

For example, an accomplished economist might find himself quite out of his depth, when he has to apply himself to work that requires less analytic and more manual skills.[9] The non-negativity constraint on the skill weights accounts for the fact that skills cannot have negative productivities - even though it is possible that some skills contribute very little, or even nothing to the productivity of an island.

Overall, this approach to labour productivity has sometimes been referred to as the "*Skill Weights Approach*" (see Laezar (2009)). The main benefit of this formulation is that it accounts for the fact that skills are sufficiently general to be transferred across occupations, whilst at the same time having different productivities in different occupations.[10]

Wages are paid per unit of skill and vary across the N occupations and M different skills, allowing for the possibility, that different skills are differentially rewarded in different occupations. In particular w_m^n is the amount paid for 1 unit of skill m from a worker currently employed in occupation n.

The competitive firm takes prices ($\{w_m^n\}_{\forall m}^{\forall n}$ and r) as given and chooses K which depreciates at rate δ, and the amount of each skill it wants to purchase from each island (S_m^n) to maximize profits Σ:

$$\max_{K,\{S_m^n\}_{\forall m}^{\forall n}} \Sigma = K^\alpha \left(\sum_{n=1}^{N} \sum_{m=1}^{M} \lambda_m^n S_m^n \right)^{1-\alpha} - (r+\delta)K - \sum_{n=1}^{N} \sum_{m=1}^{M} w_m^n S_m^n \tag{4}$$

Taking the derivative wrt. S_m^n gives:

$$\frac{\partial \Sigma}{\partial S_m^n} = (1-\alpha)\lambda_m^n K^\alpha \left(\sum_{n=1}^{N} \sum_{m=1}^{M} \lambda_m^n S_m^n \right)^{-\alpha} - w_m^n \tag{5}$$

Noting that

$$\left(\sum_{n=1}^{N} \sum_{m=1}^{M} \lambda_m^n S_m^n \right) = \sum_{n=1}^{N} l_n = L \tag{6}$$

and setting the first order condition to 0 gives us the expression for the occupation-skill specific wage:

$$w_m^n = \frac{(1-\alpha)\lambda_m^n K^\alpha}{L^\alpha} \tag{7a}$$

Which is the familiar wage equation for a Cobb-Douglas production function, scaled by the skill weight parameter λ_m^n.

[9] This all reaches back to the seminal work by Roy (1951).

[10] For example, the transferability of a skill k between two occupations x and y could be expressed by the euclidian distance $|\lambda_k^x - \lambda_k^y|$. For a more sophisticated approach see e.g. Gathmann & Schoenberg (2010).

Finally, the first order condition for the demand for capital is the familiar expression:

$$r = \alpha \left(\frac{L}{K} \right)^{1-\alpha} - \delta \tag{8}$$

2.2 Skill Supply

Time is discrete and denoted by $t = 0, 1, 2...$. The economy is populated by a continuum of infinitely lived agents, distributed on the interval $I = [0, 1]$ with measure ζ and total mass 1. Agents are ex ante homogenous and only differ with respect to the history of shocks they receive. All shocks are exogenous and take one of two forms:

1. Shocks to the skill endowment s_t that can be interpreted as events that affect a workers capacity to work in both negative and positive ways. For example a worker who suffered an accident could have his physical skills greatly reduced, while picking up a stimulating hobby might increase someone's mental abilities.

2. Shocks to the workers occupation match n_t that are interpreted as occupation transitions.[11]

Both types of shocks affect a workers productivity, albeit in differentiated ways.

In traditional treatments of income risk, shocks are typically not separated by their source, but rather by their persistence (c.f. Meghir & Pistaferri (2011)), if any distinction is made at all. In this chapter the distinction is made deliberately and serves an important function. Technological change is thought to occur at the occupation level (c.f. Autor et al. (2003)) and thus "outside" of the worker. But if all that can be observed is the evolution of labour income based on the joint movement of skills and occupation matches it is impossible to identify the contribution of technological change to earnings risk. By separating two sources of income risk, and calibrating each independently, the model is able to identify the relevant effects and therefore give a more accurate account of the effect of technological change on income and wealth inequality.

There are a finite number of M different skills, each with an associated state space $\tilde{S}^m = [0, 1]$[12]. Let us denote the Cartesian product of all these state spaces as: $\tilde{S} = \tilde{S}^1 \times ... \times \tilde{S}^M$ with associated $\sigma-$algebra $\tilde{\mathcal{S}}$.

[11] The exogeneity of occupation transitions is clearly a strong assumption, however not any stronger than the all to common assumption of exogenous income processes. There are also many "exogenous" reasons why someone might change their occupation: neccessitated by a change of location, flights of fancy...

[12] This is a necessary, but arbitrary normalization. Any finite upper bound on the state space will suffice.

Every period t, a worker $i \in I$ independently[13] draws a skill endowment $s_t = (s_{1,t}, ..., s_{M,t}) \in \tilde{S}$. In particular I assume that these draws are described by a first order Markov Process, with Transition Function $\Omega : \tilde{S} \times \tilde{\mathcal{S}} \Rightarrow [0, 1]$. Throughout the chapter I assume that this process has a *unique stationary distribution* $\bar{s}$ and impose that all draws are initiated from this stationary distribution.

The values of the workers skill endowment s_t summarize his ability to perform different tasks, with values close to 1 indicating a high proficiency and values closer to 0 indicating low skills in that area. As s_t evolves over time, an individual workers abilities change, however since $\bar{s}$ is a stationary distribution, the aggregate joint distribution remains unchanged.

The stationarity of $\bar{s}$ allows us to define the time invariant mean level of skill m across the population as:

$$\bar{\eta}_m = \int s_m \Omega(d\bar{s}) \tag{9}$$

Additionally, in every period individuals are assigned to one of the N *occupation islands*. For simplicity the process of assignment is taken to be exogenous and follows a Markov Chain of order one, summarized by the occupation transition matrix Π. Assuming that Π is *irreducible* and *aperiodic*, each stochastic process (n, Π) induces a *unique stationary distribution* $\bar{\mu} = (\bar{\mu}_1, ..., \bar{\mu}_N)$ that describes the distribution of workers across the N occupations.[14] Again I assume throughout the chapter that we are always drawing from $\bar{\mu}$ whenever necessary.

Workers supply their skills inelastically on their respective islands. Since the processes for skills and occupation transitions are independent from one another, it follows, that the total supply of skill m to occupation n is given by:

$$S_m^n = \bar{\mu}_n \bar{\eta}_m \tag{10}$$

Which is the proportion of workers on island n multiplied by the mean skill level of skill m.

This allows us to express equation (3a) as:

$$l_n = \sum_{m=1}^{M} \lambda_m^n \bar{\mu}_n \bar{\eta}_m \tag{11}$$

[13]Skill draws are independent across agents, but may be correlated across time.
[14]See for example Chapter 3.1 in Miao (2014).

Noting that $\bar{\mu}_n$ is constant across all m, we can rewrite this as:

$$l_n = \bar{\mu}_n \sum_{m=1}^{M} \lambda_m^n \bar{\eta}_m \tag{12}$$

Finally, summing over all N occupation islands, we derive the expression for the aggregate measuring labour services:

$$L = \sum_{n=1}^{N} \bar{\mu}_n \sum_{m=1}^{M} \lambda_m^n \bar{\eta}_m \tag{13}$$

This suggests that for any given transition matrix Π and skill process $(\tilde{S}, \tilde{\$}, \Omega)$, there exists a stationary distribution of skills over occupations that pins down the labour supply L and therefore the prices $\{w_m^n\}_{\forall m}^{\forall n}$ & r, for a given K. Furthermore, since the stationary distributions implied by Π and $(\tilde{S}, \tilde{\$}, \Omega)$ are unique, then so are the derived quantities and prices.

The next section describes the choices of the representative household, acting in this environment.

3 The Households Problem

Households discount the future at rate $\beta \in (0, 1)$ and inelastically supply their entire skill endowment to the *occupation island* they are assigned to. In return they receive labour income ω which they can use to purchase the consumption good c or invest in a safe asset a that pays the return r. This asset corresponds to capital that is used in production (K) and the total capital stock of the economy is given by the sum of all savings held by the households. Households can also use their existing asset stock to finance further consumption, or take on a limited amount of debt.

Financial markets in this economy are imperfect and individuals face a borrowing limit:

$$a_{t+1} \geq -\phi \tag{14}$$

Following Aiyagari (1994) the borrowing limit satisfies:

$$\phi = \left\{ \begin{array}{c} \min\left[\gamma, \frac{\omega^{\min}}{r}\right] \text{ if } r > 0 \\ \gamma \text{ if } r \leq 0 \end{array} \right\} \tag{15}$$

where $\omega^{\min}$ represents the worst possible income state.[15] In this case $\frac{\omega^{\min}}{r}$ represents the *natural debt limit* and γ is an *ad hoc* debt limit. The *natural debt limit* is based on the idea that banks will only lend up to the point where an indebted individual can repay their debt with certainty, even if they happen to draw the worst possible income realization at every point in the future. This *natural debt limit* is derived under the assumption, that the lender can confiscate all income received by the debtor for purposes of debt repayment. This is evidently a rather unrealistic assumption and is therefore rarely encountered in applications, where *ad hoc limits* are commonly used. However, the reasoning behind the *natural debt limit* gives us a strong theoretical reason for the existence of a finite lower bound on assets.

In line with this reasoning, the set of assets is defined as: $A = [-\phi, \infty)$ with associated $\sigma-$algebra $\mathcal{A}$.

I assume that the instantaneous utility function $u : [0, \infty) \Rightarrow \mathbb{R}$ is bounded, twice continuously differentiable as well as strictly increasing and strictly concave. Furthermore, it is assumed that the first derivative of u satisfies $\lim_{c\to\infty} u_c(c) = 0$ and $\lim_{c\to 0} u_c(c) = \infty$ i.e. the marginal utility derived from consumption approaches 0 as consumption approaches infinity, and vice versa. Finally, I assume that the degree of absolute risk aversion tends to 0 as consumption tends to infinity: $\lim_{c\to\infty} \inf \left(-\frac{u_{cc}}{u_c} = 0\right)$.

These restrictions are typical for both the partial equilibrium income fluctuation literature (c.f. Miao (2014)) as well as the literature on heterogeneous agents incomplete markets models with general equilibrium (c.f. Acikgoz (2018), Aiyagari (1994)).

It is well known (c.f. Aiyagari (1994), Chamberlain & Wilson (2000), Miao (2014)), that any solution to an income fluctuation problem with finite assets must satisfy $r < \left(\frac{1}{\beta} - 1\right)$. Otherwise the precautionary savings motive overrides the intertemporal substitution motive and households accumulate an infinite amount of assets. Hence I assume this condition, as well as $1 + r > 0$. The latter condition ensures that households cannot grow rich, by taking on debt, while it allows for the possibility that the interest rate might be negative. It is easy to see from the budget constraint, that - if this condition was violated - a household could increase its consumption in period $t + 1$ by taking on debt in period t: For example, let $a_t < 0$ and $1 + r < 0$, then the amount of wealth carried forward to period $t + 1$ is

[15] In our case this would be any state in which the individual has drawn $s_t = (0, ..., 0)$. In order to avoid issues with zero income, I define $\omega^{\min}$ as the income resulting from draws of s_t such that an individual that has borrowed the highest possible amount can maintain nonzero consumption by again borrowing the highest possible amount: $\omega^{\min} > r\phi$. All draws of s_t that would result in a lower labour income are assigned measure 0 by Ω.
In practice this is not going to be an issue.

$(1+r)a_t > 0$.

The exogenous state n_t describes the occupation that the worker is matched with at time t and $s_t \in \tilde{S}$ describes the workers skill set at t. Correspondingly n_t evolves according to the occupation transition matrix Π, and s_t according to the Markov Process $(\tilde{S}, \tilde{\mathcal{S}}, \Omega)$.

The households problem is as follows: Given initial conditions $(a_0, n_0, s_0) \in A \times N \times \tilde{S}$ and taking prices $r, \{w_m^n\}_{m\in M}^{n\in N}$ as given, the typical household chooses sequences of consumption $\{c_t\}_{t=1}^{\infty}$ and assets $\{a_{t+1}\}_{t=1}^{\infty}$ that solve the following sequential utility maximization problem:

$$V(a_0, n_0, s_0) = \sup_{\{c_t, a_{t+1}\}_{t=0}^{\infty}} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \tag{16a}$$

subject to the budget constraint:

$$c_t + a_{t+1} = (1+r)a_t + \omega_t \tag{17}$$

where

$$\omega_t = \sum_{n=1}^{N} \mathbf{1}_{(n=n_t)} \sum_{m=1}^{M} (w_m^n s_{m,t}) \tag{18}$$

is the total labour income derived from working in occupation n at time t. Here $\mathbf{1}_{(n=n_t)}$ is an indicator function, that takes the value 1, if the worker is employed in occupation n at time t and 0 otherwise.

To rewrite the sequential problem in recursive form, define $v(a_t, n_t, s_t; \{w_m^n\}_{\forall m}^{\forall n}, r)$ as the optimum value of the objective function, starting at the joint asset, occupation and skill set state (a_t, n_t, s_t). This gives a standard dynamic programming formulation:[16]

$$v(a_t, n_t, s_t) = \max_{a_{t+1}\geq -\phi} \left\{ u(c_t) + \beta \sum_{n_{t+1}=1}^{N} \left(\int v(a_{t+1}, n_{t+1}, s_{t+1}) \Omega(s_t, ds_{t+1}) \right) \Pi(n_t, dn_{t+1}) \right\} \tag{19}$$

For simplicity I will from now on combine the two exogenous states n_t, s_t into a single state ψ_t with associated state space $\Psi = (N \times \tilde{S})$ and $\sigma-$algebra $\mathit{\Psi}$ and transition function $\Upsilon : \Psi \times \mathit{\Psi} \Rightarrow [0,1]$. This single exogenous state now summarizes the workers occupation match, as well as his skill-set at time t. Let the combined state evolve according to the following Markov Process $(\Psi, \mathit{\Psi}, \Upsilon)$. Hence the problem can be stated as:

$$v(a_t, \psi_t) = \max_{a_{t+1}\geq -\phi} \{u(c_t) + \beta \int v(a_{t+1}, \psi_{t+1}) \Upsilon(\psi_t, d\psi_{t+1})\} \tag{20}$$

[16]Surpressing the dependance on prices for notational simplicity.

We can apply standard stochastic dynamic programming results (e.g. Theorem 9.8 in Stokey & Lucas (1989)), to conclude that $v(a_t, \psi_t)$ is strictly concave in a_t and that the policy functions: $C(a_t, \psi_t) = c_t$, $G(a_t, \psi_t) = a_{t+1}$ are continuous, single valued functions. Assumption **9.4** is met because a_t is part of the real line; **9.5** is met because $\psi_t \in \Psi$ is a countable set; **9.6** holds because $a_{t+1} \in [-\phi, (1+r)a_t + \omega_t]$; **9.7** holds by assumption; **9.10** holds by the concavity of $u(c_t)$; and **9.11** holds by the linearity of the budget constraint.

The solution to the problem can be calculated numerically by value function iteration.[17]

After solving the representative households problem, we can turn to the implied joint distribution of endogenous and exogenous variables.

Let us define a transition function $T[(a,\psi), \hat{A} \times \hat{B}] : (A \times \Psi) \times (\not{A} \times \not{\Psi}) \Rightarrow [0,1]$ for any $(a,\psi) \in \Lambda$, $\hat{A} \in \not{A}$ and $\hat{B} \in \not{\Psi}$ to be the transition function induced by the policy function $G(a_t, \psi_t)$ and the stochastic process for Ψ, with the following form:[18]

$$T[(a,\psi), \hat{A} \times \hat{B}] = \left\{ \begin{array}{c} \Upsilon(\psi, \hat{B}),\ if\ G(a_t, \psi_t) \in \hat{A} \\ 0,\ \text{otherwise} \end{array} \right\} \tag{21}$$

Let $\chi_t(\hat{A} \times \hat{B})$ be the distribution of households over the joint state space $\Lambda = A \times \Psi$ at time t. Through its definition, T provides a law of motion for the evolution of the joint state χ_t :

$$\chi_{t+1} = T\chi_t \tag{22}$$

Under the assumptions made above,[19] Acikgoz (2018) shows that the Markov process on Λ with transition function T guarantees the existence of a unique, stationary distribution $\bar{\chi}(A \times \Psi)$ that satisfies:

$$\bar{\chi} = T\bar{\chi} \tag{23}$$

It is further guaranteed, that the expected value of the assets held, using the invariant

[17] However in the practical application I solve the model with the method of endogenous gridpoints (c.f. Carroll (2006)).

[18] See Stokey & Lucas Chapter 9.6 for details.

[19] Note that this specification implies, that it is possible for the agent to rank each $\psi_t \in \Psi$ as is required for the proof to go trough: For any $\tilde{\psi} \in \Psi$ we can calculate a fixed quantity of labour income via the labour income equation (18). Applying (18) to all elements of Ψ generates a mapping from the elements of Ψ and the set of wages $\{w_m^n\}_{\forall m}^{\forall n}$ to a set $\omega^{\tilde{\psi}}$ that contains all the corresponding labour incomes. Remember, that we have defined the smallest element of this set $\omega^{\min} > 0$ when discussing the borrowing constraint. Furthermore, given the assumptions made about Ψ and λ we also know that $\omega^{\tilde{\psi}}$ has has a finite maximum $\omega^{\max}$, which is all that is needed.

distribution, is continuous in r.

4 General Equilibrium

The law of motion T can be interpreted as describing the evolution of the joint endogenous and exogenous states of a typical household over time. And as the distribution of states that are visited by the typical household over an infinitely long period of time. In line with the literature (c.f. Aiyagari (1994), Ljungqvist & Sargent (2012)) I interpret χ_t as a cross sectional distribution of households over the joint state space Λ. In particular I interpret the measure $\chi_t(\hat{A} \times \hat{B})$ as the fraction of households whose asset positions and exogenous states lie within the set $\hat{A} \times \hat{B}$ at time t.

Aggregation over this cross sectional distribution of households follows from a Strong Law of Large Numbers (Acemoglu & Jensen (2015)). In particular, following the concept of an *aggregator* described in Acemoglu & Jensen (2015) individual level uncertainty can be cancelled out by integrating over the distribution of households. This implies that all aggregate quantities are fixed and therefore non random from the perspective of the individual household.

In particular the total capital supply is given by:

$$K = \int\limits_{i \in I} a_{i,t} di \tag{24}$$

and the total labour supply by:

$$L = \sum_{n=1}^{N} \bar{\mu}_n \sum_{m=1}^{M} \lambda_m^n \bar{\eta}_m = \sum_{n=1}^{N} \bar{\mu}_n \sum_{m=1}^{M} \lambda_m^n \int_{i \in I} s_{i,m} \Omega(d\bar{s}) \tag{25}$$

Using these stationary quantities, I define a *competitive, recursive, stationary equilibrium* as follows:[20]

A value function $v(a_t, \psi_t) : \Lambda \Rightarrow \mathbb{R}$, and policy functions $C(a_t, \psi_t) : \Lambda \Rightarrow \mathbb{R}_+$ and $G(a_t, \psi_t) : \Lambda \Rightarrow A$; similarly an invariant distribution $\bar{\chi}(\hat{A} \times \hat{B})$ and a law of motion T ; an aggregate stock of capital K, a labour aggregate L; a set of wage rates $\{w_m^n(K)\}_{m \in M}^{n \in N}$ and an interest rate $r(K)$ such that:

[20] See for example Acikgoz (2018), Ljungqvist & Sargent (2012, ch. 18), Miao (2014, ch. 17).

1. The representative firm maximizes profits so that the set of $\{w_m^n\}_{\forall m}^{\forall n}$ and r solve the firms problem (4). Specifically, the set of wages $\{w_m^n\}_{\forall m}^{\forall n}$ solves the equations described by (7a) and the interest rate r solves (8).

2. Given the aggregate quantities, K and L, and prices $r, \{w_m^n\}_{\forall m}^{\forall n}$, C and G solve the households problem (20) and v solves the Bellman equation (20).

3. T is the law of motion induced by G and Υ and $\bar{\chi}$ is the stationary distribution defined by: $\bar{\chi} = T\bar{\chi}$.

4. The capital market clears: $K = \int_{i \in I} a_{i,t} di$.

Following standard arguments (e.g. Aiyagari (1994)) it can be shown that under the conditions set out above, a general equilibrium exists for this economy. Uniqueness of the stationary equilibrium is established in Acikgoz (2018).

5 Simple Example

To aid intuition I will provide a simple example of the productivity process in this section. Suppose there is only one skill h and two occupations n^{high}, n^{low} differentiated by how intensely they use the skill h. Specifically, n^{high} is associated with λ^{high} and n^{low} is associated with λ^{low}. The exogenous occupation transition matrix is given by:

	n^{high}	n^{low}
n^{high}	π_{hh}	π_{hl}
n^{low}	π_{lh}	π_{ll}

with an associated stationary distribution $\bar{\mu} = (\bar{\mu}^{high}, \bar{\mu}^{low})$.

The skill h can take on two values: h^{high}, h^{low} with transition probabilities:

	h^{high}	h^{low}
h^{high}	ϖ_{hh}	ϖ_{hl}
h^{low}	ϖ_{lh}	ϖ_{ll}

with an associated stationary distribution $\bar{s} = (\bar{s}^{high}, \bar{s}^{low})$.

The joint state space Ψ is given by the set

$\{(\lambda^{high}, h^{high}), (\lambda^{high}, h^{low}), (\lambda^{low}, h^{high}), (\lambda^{low}, h^{low})\}$ with transition matrix Υ:

	$(\lambda^{high}, h^{high})$	$(\lambda^{high}, h^{low})$	$(\lambda^{low}, h^{high})$	(λ^{low}, h^{low})
$(\lambda^{high}, h^{high})$	$\pi_{hh}\varpi_{hh}$	$\pi_{hh}\varpi_{hl}$	$\pi_{hl}\varpi_{hh}$	$\pi_{hl}\varpi_{hl}$
$(\lambda^{high}, h^{low})$	$\pi_{hh}\varpi_{lh}$	$\pi_{hh}\varpi_{ll}$	$\pi_{hl}\varpi_{lh}$	$\pi_{hl}\varpi_{ll}$
$(\lambda^{low}, h^{high})$	$\pi_{lh}\varpi_{hh}$	$\pi_{lh}\varpi_{hl}$	$\pi_{ll}\varpi_{hh}$	$\pi_{ll}\varpi_{hl}$
(λ^{low}, h^{low})	$\pi_{lh}\varpi_{lh}$	$\pi_{lh}\varpi_{ll}$	$\pi_{ll}\varpi_{lh}$	$\pi_{ll}\varpi_{ll}$

and associated stationary distribution $\varrho = (\bar{\mu}^{high}\bar{s}^{high}, \bar{\mu}^{high}\bar{s}^{low}, \bar{\mu}^{low}\bar{s}^{high}, \bar{\mu}^{low}\bar{s}^{low})$.

The labour supply in the first occupation l^{high} is therefore given by:

$$l^{high} = \left(\left(\bar{\mu}^{high}\bar{s}^{high} * \lambda^{high} * h^{high}\right) + \left(\bar{\mu}^{high}\bar{s}^{low} * \lambda^{high} * h^{low}\right)\right)$$

with l^{low} similarly defined:

$$l^{low} = \left(\left(\bar{\mu}^{low}\bar{s}^{high} * \lambda^{low} * h^{high}\right) + \left(\bar{\mu}^{low}\bar{s}^{low} * \lambda^{low} * h^{low}\right)\right)$$

Total labour supply is given by:

$$L = l^{high} + l^{low} \tag{26}$$

Given L one can easily derive the set of occupation specific wage rates w^{high} and w^{low} for any value of the aggregate capital stock K:

$$w^{high} = \frac{(1-\alpha)\lambda^{high}K^{\alpha}}{L^{\alpha}} \tag{27}$$

$$w^{low} = \frac{(1-\alpha)\lambda^{low}K^{\alpha}}{L^{\alpha}} \tag{28}$$

Taking these as given the worker knows his labour income in each of the 4 states $\psi \in \Psi$. Taking these as given one can solve the typical households problem and the rest of the model in the familiar fashion.

After illustrating how the aggregate Labour Supply is derived in this setting let us turn to the properties of the implied income process. By comparing the *structural* income process defined by the model with a reduced form version, I will highlight some of the benefits derived from thinking about the structure of income determination in the given task-skill framework. Most HIM models can be calibrated using information about the persistence and unconditional variance of income. For simplicity, I will focus on the unconditional variance, but similar arguments can be made using the persistence.

Average labour income in this economy is given by:

$$E(\omega) = \begin{bmatrix} (\bar{\mu}^{high}\bar{s}^{high}w^{high}h^{high}) + (\bar{\mu}^{high}\bar{s}^{low}w^{high}h^{low}) \\ +(\bar{\mu}^{low}\bar{s}^{high}w^{low}h^{high}) + (\bar{\mu}^{low}\bar{s}^{low}w^{low}h^{low}) \end{bmatrix} \tag{29}$$

The variance of labour income is given by:

$$Var(\omega) = \begin{bmatrix} \bar{\mu}^{high}\bar{s}^{high} * (w^{high}h^{high})^2 + \bar{\mu}^{high}\bar{s}^{low} * (w^{high}h^{low})^2 \\ +\bar{\mu}^{low}\bar{s}^{high} * (w^{low}h^{high})^2 + \bar{\mu}^{low}\bar{s}^{low} * (w^{low}h^{low})^2 \end{bmatrix} - E(\omega)^2 \tag{30}$$

Now suppose that the econometrician is unable to observe changes to an individuals skill-set or occupation match, but instead only observes total labour income $y = w * h$ in the 4 different income states. Denote these states (in order of appearance above) by $y_1, .., y_4$ and $p_{11}, ..., p_{44}$ as the associated transition probabilities between the states.[21]

Together, $y = \{y_1, .., y_4\}$ and the transition matrix P defined by $p_{11}, ..., p_{44}$ describe a reduced form income process that will be familiar from many standard HIM applications that utilize discrete state Markov chain approximations to autoregressive income processes (c.f. Aiyagari (1994), Rouwenhorst (1995), Tauchen (1986)).

In particular, letting $\bar{p}_n$ denote the unconditional probability of being in income state $n = \{1, .., 4\}$.

Average labour income given by the reduced form approach is:

$$E(y) = (\bar{p}_1 y_1 + \bar{p}_2 y_2 + \bar{p}_3 y_3 + \bar{p}_4 y_4) \tag{31}$$

And the variance is given by:

$$Var(y) = \left(\bar{p}_1 y_1^2 + \bar{p}_2 y_2^2 + \bar{p}_3 y_3^2 + \bar{p}_4 y_4^2\right) - E(y)^2 \tag{32}$$

Comparing equation (29) with (31) and (30) with (32) we can spot the problem, that this approach is trying to address. If we are able to observe the reduced from labour income states, we should expect both approaches to deliver the same results. However, if we were trying to anticipate changes to the two moments of the income distribution, we would

[21] Every p is a combination of π and ϖ.

struggle to find a solid basis for changes to the parameters of reduced form income process. Without grounding the determinants of income in any structural relationship, forecasting the evolution of any moment of the income distribution will involve a degree of arbitrariness.

Focussing on the determination of labour income through $y = w(\lambda) * h$[22] we can see that changes to observed labour income y can be driven by both, technological factors - as embodied in $w(\lambda)$ - and changes in the distribution of human capital. The former effect might be dominant throughout a period of rapid technological change, whilst the latter could be the driving force after a major educational reform.

The same can be said of the transitions between the different income states $p = \pi * \varpi$ where π might represent structural behavior in the labour market and ϖ the speed at which individuals change their skill-set. In both cases the two principal actors of Tinbergen's *race* interact to produce one indivisible outcome.

The rest of the chapter essentially focusses on addressing this issue. Exploiting the fact that the structural and reduced from income processes should produce the same results, I am able to calibrate the structural model with observable data. Then I can use changes to the structural environment to generate changes in the income process that are based in popular theories on technological change in order to observe the effects on income and wealth inequality.

The next section describes the calibration of the structural model in a stationary environment, setting the stage for the policy experiments in the following sections.

6 Calibration

6.1 Introduction

I calibrate a stationary version of the model using data from 8 waves of the Understanding Society (USoc.) survey, covering the years 2009 - 2018. Unless differently stated the analysis focuses on white British males, aged between 25 and 55, who are employed for a number of consecutive periods.[23]

The sample is selected to represent the closest approximation to a truly competitive labour market, in which observed pay is most closely related to actual productivity. This is

[22] Here I made the dependence of w on λ explicit.

[23] So if I observe some individual as employed for a number of periods who becomes unemployed after a certain period I drop all further observations on this individual.

in line with much of the literature on labour market outcomes. As a measure of income I use the usual net pay per month in the current job, deflated by the CPI. In order to avoid issues with top (bottom) coded incomes I trim the top and bottom 0.5% in terms of labour income.[24] As any survey, Understanding Society contains a number of accidental errors that occur along the way from interview to finished dataset. For example in a number of individuals are reported to earn £1 per month which can be safely attributed to data error. By removing the worst outliers I ensure that the estimates are not disturbed by too much random noise.

6.2 Skill Measures & Processes

For this application I limit myself to two types of general skills: Cognitive and Physical.[25] These can be seen as embodiments of the two pillars of human capital theory, with the former representing intellectual capacity, and the latter bodily health and physical ability. Apart from this, the distinction between analytic and manual tasks has long been at the heart of the literature on task-biased technological change (c.f. Autor et al. (2003)). In line with the Task-Skill framework, I suggest that cognitive skills will be applied to cognitive tasks - i.e. there is a skill *Cognitive* amongst the $M = 2$ skill types, and a set of corresponding cognitive skill weights: $\left\{\lambda^n_{Cog}\right\}_{n=1}^N \in \lambda$. And similarly for Physical skills.

Using test scores for a variety of numerical and verbal intelligence tests in wave 3 of the USoc, I generate a composite measure of cognitive ability by performing a Principal Component Analysis and selecting the first principal component. The method here follows Whitley et al. (2016), who use the same survey to study the effect of aging on cognitive decline.[26]

The USoc surveys also include the SF-12 Physical Component Summary (PCS) Index, which is a composite index evaluating the physical health of an individual (c.f. Ware et al. (2001)).[27] This is a self completed questionnaire that encompasses 12 questions regarding different aspects of physical health and fitness. The PCS score is then derived from the an-

[24] Choosing 0.25% as cutoffs does not significantly alter the results. As does not excluding any observations.

[25] At this point it appears useful to remind ourselves, that there a many possible ways of dividing the complex manifold of Tasks and Skills. One could have split the workers into "skilled" and "unskilled" workers, potentially employing some sort of educational qualification as dividing characteristic. Alternatively one could have employed the complex-routine/manual-cognitive distinction first used by (Autor et al. (2003)). Ultimately the final choice comes down to preference and data availability.

[26] For more information on this procedure see the Appendix.

[27] Guvenen et al. (2017) use this index to proxy physical ability.

swers, providing a summary statistic rating the respondents health. For lack of alternatives I take this as a proxy for an individuals physical ability (c.f. Lise & Postal-Viney (2017)). I standardize both scores on the interval [0,1] in order to make them comparable.

The cross sectional correlation between both measures is around 0.10, so I believe that I am justified in treating both processes as independent.

For simplicity the evolution of both skills are defined as stationary AR(1) processes in logarithms:

$$\log(s_{m,t+1}) = \rho_m \log(s_{m,t}) + \varepsilon^m_{t+1} \tag{33a}$$

$$|\rho_m| < 1 \tag{33b}$$

$$\varepsilon^m_t \sim N(0, \sigma^2_m) \tag{33c}$$

Their calibration will provide an important part of the income dynamics faced by the workers. I use the availability of multiple measures of the SF-12 PCS in the data (the test is completed every year) to estimate the following regression equation:

$$\log(s^{Phy}_{t+1}) = \rho_{Phy} \log(s^{Phy}_t) + \varepsilon^{Phy}_{t+1} \tag{34a}$$

$$\varepsilon^{Phy}_t \sim N(0, \sigma^2_{Phy}) \tag{34b}$$

Unfortunately a similar approach is not possible for the measure of cognitive ability, since I only have a single observation per individual.[28] I therefore, chose the relevant parameters to match key moments in the data.[29]

The table below summarizes the relevant parameters:

	Cognitive	Physical
$\overline{\eta}$	0.681	0.685
ρ	0.867^+	0.909
σ_ε	0.245^+	0.154
	+	calibrated

Table 1: Skill Processes

[28] According to USoc, another cognitive skills test is planned in wave 9, which might provide a possibility for checking the validity of my calibration.

[29] The target moments are provided by targeting the reduced form log-normal income process in the data, namely the persistence of the AR(1) process, the standard deviation of innovations and the standard deviation of log(Income). For the calibration procedure, see Appendix.

The pattern that emerges from these values seems reasonably intuitive. Both processes have a high level of persistence with a standard deviation of innovations between 15 and 25%. Unfortunately there is - to the best of my knowledge - no study that approaches the evolution of skills in a similar manner, and therefore it is impossible to verify whether these calibrations are reasonable. The closest comparison study might be Huggett et al. (2011) who estimate a standard deviation of shocks to human capital of around 11%. Since shocks to human capital in this model are composed of shocks to either skills, as well as occupation transitions, the values do not seem too far off the mark.

For the computational implementation I approximate each (33a) by a 5 state discrete Markov Chain using the Rouwenhorst (1995) method.[30]

6.3 Skill Weights

I now explain how to recover the skill weights λ from the data. Many recent papers attempt to estimate the return to specific work activities - usually referred to as "task returns", which play a similar role as the skill weights referred to here - by using survey information on the importance of certain work tasks (c.f. Autor & Handel (2013), Baumgarten et al. (2013), Firpo et al. (2011)). These approaches usually have to deal with selection bias, where individuals with high unobserved ability in certain tasks, sort into occupations where this task is used very intensely according to a Roy framework (c.f. Autor & Handel (2013), Firpo et al. (2011), Roys & Taber (2016)). This means that if skill levels are unobserved, estimated task returns will be biased upwards under positive sorting. The main benefit of the route take here is that skills are observed and can thus be controlled for.[31]

Recall that an individual i, in occupation n earns labour income $\omega_{i,n}$[32] equal to:

$$\omega_{i,n} = \sum_{m=1}^{M} (w_m^n s_{i,m}) \tag{35}$$

substituting the wage equation (7a) into this equation gives:

$$\omega_{i,n} = \sum_{m=1}^{M} \left(\frac{(1-\alpha)\lambda_m^n K^\alpha}{L^\alpha} \right) s_{i,m} \tag{36}$$

noting that some of these terms are invariant across all m we can rewrite this equation as:

[30] The discretization error resulting from this procedure is of the order of 5%.
[31] Regressing wages on skills is the procedure suggested by Autor (2013).
[32] For expositional clarity, I make the dependance of income on the occupation match n explicit.

$$\omega_{i,n} = \underbrace{(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}}_{\text{common}} \underbrace{\sum_{m=1}^{M} \lambda_m^n s_{i,m}}_{\text{individual}} \tag{37}$$

It is easy to see that the above equation can be split into a common part $(1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$ that is constant across all individuals in the economy, and a worker specific part $\sum_{m=1}^{M} \lambda_m^n s_{i,m}$ that depends on the workers individual skill-set and occupation match.

Our estimation will focus on the individual specific part. For the econometric specification I will treat the common part as a constant $\Xi = (1-\alpha)\left(\frac{K}{L}\right)^{\alpha}$ that multiplies each $\lambda_m^n \in \lambda$. Hence I will redefine the skill weights to include the level information included in the common constant:

$$\tilde{\lambda}_m^n = \Xi * \lambda_m^n \tag{38}$$

Using this transformation we derive the following expression:

$$\omega_{i,n} = \sum_{m=1}^{M} \tilde{\lambda}_m^n s_{i,m} \tag{39}$$

Since $\omega_{i,n}$ and $s_{i,m}$ are observed in the data, I can estimate (39) for a given n, and obtain estimates of $\tilde{\lambda}^n$ as the coefficients of a simple OLS regression. Furthermore I can pool all n and use dummy variables to estimate the whole set of $\tilde{\lambda}$ jointly:

$$\omega_{i,n} = \sum_{n=1}^{N} \left\{ D_n \sum_{m=1}^{M} \tilde{\lambda}_m^n s_{i,m} \right\} \tag{40}$$

Here D_n is an occupation dummy that is equal to 1 if the individual is employed in occupation n and zero otherwise. I obtain estimates of the rescaled skill weights $\tilde{\lambda}_m^n$ as the coefficients of the measured skills $s_{i,m}$ interacted with the occupation dummies D_n. This econometric strategy draws on Deming (2017) and Gensowski (2017) who investigate the return to different skills using similar reduced form specifications.

The results of this regression for all 9 major SOC2000 occupations are reported below:

	Labour Income
Managers X Cognitive Skills	2136.867***
	(237.272)
Professionals X Cognitive Skills	2012.366***
	(305.273)
Associate Professional X Cognitive Skills	2169.107***
	(242.370)
Administrative X Cognitive Skills	1635.570***
	(259.414)
Skilled Trades X Cognitive Skills	1358.362***
	(207.406)
Personal Services X Cognitive Skills	1096.985***
	(308.619)
Sales X Cognitive Skills	1407.666***
	(334.081)
Plant & Machine Operatives X Cognitive Skills	1338.344***
	(159.251)
Misc X Cognitive Skills	998.316***
	(148.150)
Managers X Physical Skills	1327.115***
	(239.439)
Professionals X Physical Skills	1416.625***
	(314.107)
Associate Professional X Physical Skills	799.192***
	(241.528)
Administrative X Physical Skills	814.155**
	(261.155)
Skilled Trades X Physical Skillsl	1336.246***
	(194.737)
Personal Services X Physical Skills	1072.406***
	(296.482)
Sales X Physical Skills	530.833
	(288.284)
Plant & Machine Operatives X Physical Skills	1254.784***
	(146.051)
Misc X Physical Skills	1109.200***
	(134.661)
R-squared	0.878
N	4393

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 2: Skill Weights Regression - Major SOC2000 Occupations

In order to reduce the number of variables, I separate the 9 major occupational groups into three larger occupation clusters. For this I run a hierarchical clustering algorithm on the skill weights obtained from this initial regression. The clustering algorithm groups the occupations according to their similarity with respect to the estimated skill weights. The resulting clusters[33] loosely represent some of the key worker types in the skill and task biased technological change literature:

(1) Managers, Professionals & Associate Professionals, representing traditional high skilled occupations that are intensive in complex analytic tasks.

(2) Administrative & Sales occupations, represent middle to low skilled occupations with a focus on routine cognitive tasks.

Finally (3), Skilled Trades, Personal Services, Plant & Machine Operatives & Miscellaneous occupations, cover workers that work in manual tasks.[34]

The clear interpretability of the different clusters already suggests, that the occupation specific skill weights $\tilde{\lambda}^n$ that I have obtained from the regression pick up information about the structural differences between the different occupations. To analyze this further, I repeat the regression analysis with the reduced set of occupations.

	Labour Income
Managers & Professionals X Cognitive Skills	2134.534***
	(151.876)
Admin & Sales X Cognitive Skills	1692.235***
	(214.909)
Skilled Workers, Services, Operatives & Misc X Cognitive Skills	1273.780***
	(97.094)
Managers & Professionals X Physical Skills	1172.172***
	(153.777)
Admin & Sales X Physical Skills	593.662**
	(207.421)
Skilled Trades, Services, Operatives & Misc X Physical Skills	1217.692***
	(89.798)
R-squared	0.873
N	4393

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 3: Skill Weights Regression - Occupation Clusters

[33] I use single linkage, average linkage and Ward's linkage, all of which result in the reported clustering.

[34] In order to save space I will sometimes refer to these groups by abbreviations; e.g. "Admin" for "Administrative & Sales".

The regression table shows clear, discernible differences in the returns to cognitive and physical skills across the three occupation clusters. In order to aid further interpretation, I rescale the estimated skill weights, so that the implied total labour supply is standardized to 1. The results are shown below:

	Managers & Professionals	Admin & Sales	Trades, Services, Operators & Misc.
$\lambda^{Cognitive}$	1.04	0.82	0.62
$\lambda^{Physical}$	0.57	0.29	0.59
$\frac{\lambda^{Cognitive}}{\lambda^{Physical}}$	1.82	2.85	1.05

Table 4: Standardized Skill Weights

Taken at face value the results are quite revealing by themselves: Cognitive skills seem to be associated with higher productivities through the board, with a reasonable ranking of the cognitive skill weights across the different occupations. Managers, Professionals and Associate Professionals tend to have higher returns to their cognitive skills than the other occupations, with Skilled Workers, Services, Machine Operators & Misc. exhibiting a sizeable 40% gap to the Managers & Professionals group.

Curiously Managers, Professionals and Associate Professionals also have relatively high returns to Physical skills, which however might reflect some higher general level of productivity that is unobserved. My preferred interpretation of this observation is that Physical skills are less proxies for physical strength, and more indicative of stamina and resilience. Given that many managerial and professional occupations can be very demanding and stressful, it should not be surprising that the returns to physical health are high.

Setting this issue aside for the moment, I also confirm that Skilled workers have high returns to Physical skills - in particular when compared to their cognitive returns. Administrative & Services meanwhile have the lowest returns to Physical skills both in absolute and in relative terms.

The ratio of the cognitive and the physical skill weight, provides us with an indication about the degree of skill risk faced by each occupation: Since skill shocks are independent, occupations with a high ratio are much more exposed to fluctuations in labour income due to high or low realizations of the cognitive skill component. Administrative and Sales workers for example seem to be very specialized in cognitive tasks, with a skill weight over three times higher than the physical skill weight. This implies that they are much more exposed to changes in their cognitive skills and, correspondingly face higher within group income inequality. This stands in stark contrast to the more balanced Skilled worker types, whose occupations place about the same weight onto both types of work activities. Correspondingly we should expect these to be less exposed to income risk due to skill fluctuations, and also exhibit lower within group income inequality.

Overall the results seem to confirm a picture where the returns to different skills are differentiated across different occupations, with a somewhat clear pattern of task specialization.

6.4 Occupation Transitions

I estimate the yearly[35] job-to-job transition probabilities from the data.

	Managers	Admin	Trades
Managers	97.92	0.79	1.29
Admin	8.21	88.12	3.67
Trades	2.92	0.86	96.22
Total	58.22	9.39	32.39

Table 5: Cluster Transition Probabilities (Percentages)

As can be seen from the estimated occupation transition matrix, transitions between occupational clusters are relatively rare. Even for the Administrative & Sales cluster, the probability to stay in the same cluster year by year is 88%. The low turnover can probably be attributed to two factors:

1. It is well known, that individuals are more likely to change their occupation after an unemployment spell (c.f. Carrillo-Tudela et al. (2016)), however I have excluded unemployed individuals from the sample.

2. Gathmann & Schoenberg (2010) show, that occupation transitions are more likely to occur between occupations that have similar task content. Since the occupational clusters have been constructed based on similarity of task content, it is very likely that the level of aggregation masks occupation transitions occurring between the component occupational groups of each cluster.

[35] Technically these are wave - wave transitions, but in practice these almost perfectly line up as USoc tries to interview with a yearly rythm.

6.5 Other Parameters

I set the functional form of the utility function to $CRRA$ and calibrate the coefficient of relative risk aversion $\varkappa = 2$, and the time preference $\beta = 0.975$, which yields a risk free interest rate of 2.56%. In line with most calibrations I set $\alpha = 0.36$ and the rate of depreciation to $\delta = 0.1$. I set the ad hoc borrowing limit $\gamma = 0$. For the final calibration I standardize the occupation specific productivities so that the total Labour Supply is equal to 1.

7 The Stationary Model

7.1 Income - Overall Sample

Since the aim of this model is to put some structure on the earnings process, it will be important to evaluate the model generated income process against the dynamics found in the data. As a benchmark, I use a reduced form earnings process of the form:

$$\log(y_{i,t+1}) = \kappa \log(y_{i,t}) + \varepsilon_{i,t+1} \tag{41a}$$

$$|\kappa| < 1 \tag{41b}$$

$$\varepsilon_{i,t} \sim N(0, \sigma_\varepsilon^2) \tag{41c}$$

I estimate the parameters for κ and σ_ε from the USoc data and use them to calibrate the productivity process in the model. After solving the model I simulate 5000 individual labour market histories, each for 40 periods in order to obtain the relevant parameter estimates. The table below shows the estimated parameters of the income process for the data and the models:

	κ	σ_ε	σ_y	Gini	$\frac{Mean}{Median}$
Data	0.882	0.155	0.399	0.23	1.09
Model	0.879	0.173	0.388	0.22	1.09

Table 6: Income Process

Comparing the parameter estimates from the model with the ones from the data immediately highlights what a good job the model does at reproducing the patterns in the data. The persistence and the standard deviation of income idiosyncratic income shocks are fairly well matched. Across the cross sectional dimension, the model matches the data closely.

7.2 Income - Within and between Occupations

A main advantage of the model is that we can attribute meaningful interpretations to the different productivity states: namely, they correspond to employment in different occupations. Within- and between occupation income inequality has recently become a topic of interest for macroeconomists (c.f. Kambourov & Manovski (2009)). The following table compares the within- and between occupation income inequality produced by the model with that observed in the data.[36]

	$\bar{y}_{Data}$	$\bar{y}_{Model}$	$\bar{y}_{Data} - \bar{y}_{Model}$	$\frac{\sigma^y_{Data}}{\sigma^y_{Model}}$	$Gini_{Data}$	$Gini_{Model}$
Managers	0.16	0.13	0.03	1.04	0.20	0.20
Admin	-0.23	-0.24	0.02	0.86	0.20	0.23
Trades	-0.19	-0.14	-0.05	1.02	0.19	0.18

Table 7: Between and Within Occupation Income Inequality

It can be seen that the model is able to capture the relative ranking of the different occupations by their mean income fairly well and does a decent job at capturing some of the between group inequality observed in the data. It only slightly overshoots with respect to the mean income of the Skilled, Services, Operatives & Misc. cluster.

Further, the model does a decent job with respect to within group inequality, both measured by the within group standard deviation of y as well as the Gini coefficient. In general, however the between group inequality tends to be between a little higher in the data than what the model can reproduce. This should not be too surprising, as the model does not allow for selection into preferred occupations, or account for any source of ex ante skill heterogeneity. Nonetheless, the model produces a surprisingly accurate picture of between- and within occupation income inequality, suggesting that the model has some merit.[37]

7.3 Assets

Having established that the model is able to reproduce important features of the data with respect to the income process, let us turn to the quantitative predictions of the model with respect to assets. Unfortunately, the USoc. survey does not contain much information on

[36] Here y is the logarithm of income. A bar indicates the average.
[37] In particular when one consideres that these moments were not directly targeted by the calibration.

	Model		Standard Aiyagari		WAS*	
	Mean	Gini	Mean	Gini	Mean	Gini
Overall	1	0.51	1.03	0.50	1**	0.62
Managers	1.14	0.50	-	-	1.41	0.57
Admin	0.81	0.58	-	-	0.75	0.66
Trades	0.73	0.58	-	-	0.53	0.63

* total personal wealth ** standardized to allow relative group comparisions

Table 8: Wealth Inequality

asset holdings, so that a comparison with the empirical distribution is not possible. Instead I compare the results to some measures of wealth inequality derived from the Wealth and Assets (WAS) survey,[38] as well as a standard Aiyagari (1994) model that has been calibrated using a 12 state discrete Markov chain representation of the income process (41a).[39] This has long been an established approach in the literature, and should provide us with a solid comparison for the model's predictions.

The table below summarizes some important features of the asset distribution implied by the model, the WAS, as well as the baseline Aiyagari:

Quantitatively the predictions borne out by the baseline Aiyagari model and the one with skills and occupations appear to be quite similar. There appears to be a slightly higher level of savings - and conversely lower level of wealth inequality - in the baseline model, but the differences are only minor. It is reassuring to see the similarity between both models, as it suggests that the model does not generate any "unusual" asset dynamics.

Overall it is notable that the implied wealth inequality is far below empirical estimates. This is particularly evident when looking at the values of the Gini coefficient: the values here range around 0.5, whilst the empirical esimate is 0.62. It is important to emphasize that this is potentially due to the fact that the income process does not take into account all sources of income variation, but rather focusses on income inequality due to differences in human capital. However, the model does a decent job at capturing the between and within group wealth inequalities, even though it doesn't quite reproduce quite so extreme disparities, as the WAS suggests. Managers hold, on average about 50% more wealth than the other occupational groups, which have a similar level of mean wealth. Conversely, within group wealth inequality is about 8 Gini points higher within the latter two groups.

[38] Waves 3 & 4.

[39] For this approximation I use Rouwenhorst (1995).

The graph below shows the stationary distribution of assets. As expected, the patterns described by both models are almost indistinguishable. The takeaway from this exercise is that the models implications for wealth inequality are very close to those obtained from the standard model. Having established the properties of the stationary model, I will use it to perform several pieces of policy relevant analysis in the following sections.

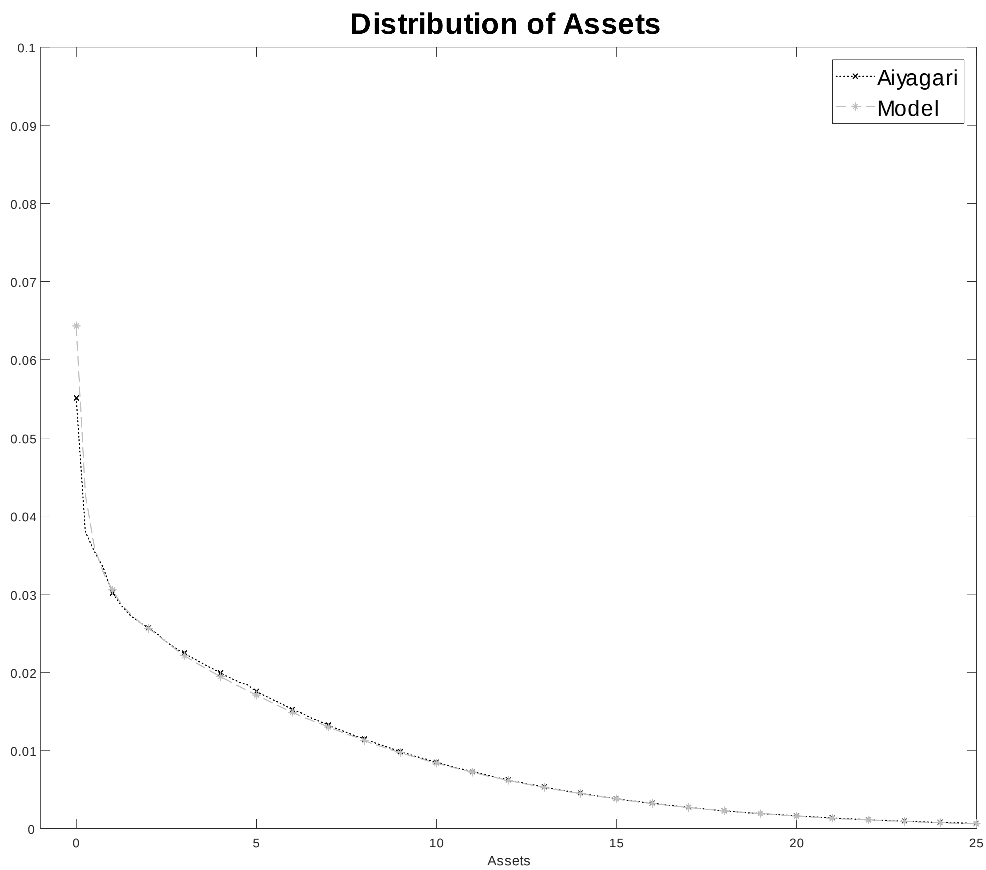


Figure 1: Stationary Distribution of Assets

8 Income Risk Decomposition

The key difference between the economic model presented in the last sections and the standard Aiyagari (1994) model, is that the former decomposes the variation of an individual workers income into two parts: 1. Variation in the amount of skills that the worker can supply, and 2. Changes in the demand for the skills of the individual worker due to occupational transitions. Both aspects are relevant to understanding income fluctuations, but are representing fundamentally different parts of economic life.

In the following section I will utilize this decomposition, to analyze the effect of each aspect on wealth & consumption inequality.

The skill supply in this model is governed by the autoregressive skill process (33a), which determines the stock of cognitive and physical skills available to the individual worker. My preferred interpretation is, that shocks to the individual's skillset represent life events that are at least somewhat exogenous. This is quite evident with respect to physical skills, where accidents and illness might play the role of exogenous shocks, but similarly a sudden training opportunity might be reflected in a positive cognitive skill shock.

Adopting this interpretation, the standard deviation of skill shocks becomes a measure of inequality of opportunity. A high spread of shocks to physical skills for example suggests that some individuals have better access to healthcare facilities than others, whilst a low spread of innovations to cognitive skills would e.g. suggest and approximately equitable distribution of educational opportunities.

Similarly, the set of skill weights, summarizes the productivity (and thus demand for) differentiated skills. As technology evolves these values are liable to change, posing a source of considerable risk for workers, who might find that their skills are no longer in demand.[40]

To assess the differentiated impact of these different sources of risk might be of interest to a variety of policymakers, and being able to provide an analysis at this disaggregated level is one of the key advantages of the model presented here.

In particular, a government might try to enact legislation in order to affect the risk posed by these different channels. These could be reforms to the healthcare of educational system, acting as insurance against physical and cognitive shocks respectively. Similarly, a government might try to encourage some type of technological progress, by imposing suitable

[40] More on this topic in the next section.

regulations on different industries,[41] thereby affecting the demand for different skills.

The structure of the model provides an opportunity, to evaluate the the impact of these different policies on the mean and distribution of income, consumption and wealth.

The following tables present a set of comparative statics exercises, investigating the effect of changes to the risk posed by cognitive and physical skill shocks, as well as changing technological demands.

Holding all other parameters constant, I vary the size of σ_{Cog}, σ_{Phy}, λ^{Cog} & λ^{Phy} and solve the model for each specification, obtaining the stationary distribution of labour income, wealth & consumption in each case. I assume, that the *ceteris paribus* assumption will hold in the short to medium run and that the approximation is accurate close to the calibrated stationary model.[42]

The values reported in the tables are elasticities, calculated as the percentage change of the level of the variable of interest, relative to its baseline value, divided by a percentage change in risk.

Labour Income

	Mean				Inequality			
	σ_{Cog}	σ_{Phy}	λ_{Cog}	λ_{Phy}	σ_{Cog}	σ_{Phy}	λ_{Cog}	λ_{Phy}
Overall	0.22	0.07	0.68	0.39	0.71	0.14	0.25	-0.11
Managers	0.23	0.07	0.71	0.36	0.86	0.13	0.25	-0.11
Admin	0.25	0.05	0.79	0.26	0.92	0.05	0.20	-0.14
Trades	0.20	0.08	0.58	0.51	0.68	0.33	0.29	0.05

Table 9: Risk Elasticities - Labour Income

Consumption

	Mean				Inequality			
	σ_{Cog}	σ_{Phy}	λ_{Cog}	λ_{Phy}	σ_{Cog}	σ_{Phy}	λ_{Cog}	λ_{Phy}
Overall	0.16	0.05	0.66	0.40	0.58	0.12	0.22	-0.10
Managers	0.16	0.05	0.68	0.38	0.76	0.14	0.20	-0.05
Admin	0.14	0.05	0.71	0.34	0.59	0.04	0.04	0.00
Trades	0.15	0.06	0.59	0.47	0.48	0.19	0.25	-0.06

Table 10: Risk Elasticities - Consumption

[41] See for example the suggestions in Atkinson (2015).

[42] i.e. a small change in $\lambda^{Cog}/\lambda^{Phy}$ will not cause a major change in the allocation of workers across occupations, and a small change in $\sigma_{Cog}/\sigma_{Phy}$ will not drastically change the mean or persistence of the skill process.

	Wealth							
	Mean				Inequality			
	σ_{Cog}	σ_{Phy}	λ_{Cog}	λ_{Phy}	σ_{Cog}	σ_{Phy}	λ_{Cog}	λ_{Phy}
Overall	0.21	0.05	0.69	0.36	0.16	-0.05	0.03	-0.07
Managers	0.20	0.03	0.71	0.32	0.22	-0.04	0.03	-0.05
Admin	0.30	0.03	0.77	0.26	0.08	-0.04	-0.03	-0.01
Trades	0.21	0.13	0.61	0.53	0.07	-0.08	0.04	-0.11

Table 11: Risk Elasticities - Wealth

An analysis of these elasticities, suggests that overall the cognitive skill component of risk (σ_{Cog} & λ^{Cog}) tends to have a bigger impact on aggregate outcomes than corresponding changes in the domain of physical skills. Furthermore changes in technology (λ^{Cog}& λ^{Phy}), tend to have a larger impact on the mean of assets or consumption than on the corrsponding Gini coefficient. Whilst the opposite is true for changes in skill risk (σ_{Cog} & σ_{Phy}).

Overall, the mean-elasticities of all three variables with respect to all sources of risk appears to be similar in magnitude, however when it comes to inequality, wealth appears to be much less responsive than both income and consumption. This suggests that none of the suggestesd policy channels will be very effective in addressing wealth inequality, even though they might be effective against income inequality.

The decomposition allows for some interesting thought experiments. For example, by enacting legislation, to reduce σ_{Cog} (e.g. a schooling reform), a government could effectively inequality along all dimensions, but at the same time, the model predicts that the very same policy will cause a reduction in overall savings and therefore hurt production, incomes and ultimately consumption. Uncovering unintended consequences such as these might be one of the potential applications of this model.

Interestingly, encouraging technologies that make productive use of physical skills, has the potential of reducing inequality overall, whilst also increasing income, savings & consumption.

This section has focussed on assessing the impact of different sources of risk on income, wealth and consumption. In the next section I will focus on the effect of technological change on the economy.

9 Cognitive Skill Biased Technological Change

9.1 Heterogenous Technological Change

The following presents a simple application that exemplifies the usefulness of the model to study the impact of technological change on the income and wealth distribution. In particular, I investigate the effects of cognitive skill biased technological change over the period 1980 - 2016 in the UK, using the model and see if it can capture the trends that we observe in the data.

A growing number of papers has documented changes in the demand for (and the return to) different work tasks.[43] The general consensus falls on the side of a rising demand for non-routine/complex/analytical/cognitive skills for the period of the 1980s-2000s (c.f. Autor et al. (2003), Spitz-Oener (2006)) with a potential tampering off since (c.f. Beaudry et al. (2016)). Conversely, the demand for routine/manual tasks has been falling throughout this period. These general trends are usually attributed to the rising importance of information & communication technologies which complement the former and displace the latter. These changes can have substantial effects for income inequality. For example, in a recent paper Burstein et al. (2019) argue that computerization can account for as much as 80% of the rise in the skill premium in the US between 1983 and 2003.

Although the computerization of the workplace began in the 1980's and the internet started to permeate into the wider economy in the early 2000's, it is clear to many that the information age is merely in its infancy. New technologies, such as artificial intelligence, big data and machine learning are set to radically transform all sectors of the economy. Given the economic insecurity and potential social upheaval, associated with the move to the *Economy 4.0,* it is of great importance to model and study the impact of technological change on the distribution of income and wealth.

Since Krussell & Smith (1998) technological change has also been an important research area in the heterogenous agent literature (c.f. Heathcote et al. (2009) for a survey). This literature has traditionally studied the effect of aggregate uncertainty created by shocks to total factor productivity. Adding aggregate productivity shocks, however does little to help us understand the potential effects from task biased technological change. This is because these affect all workers uniformly, whilst the central finding associated with the research of

[43] See for example: Autor et al. (2003), Autor & Price (2013), Firpo et al. (2011), Rohrbach-Schmidt & Tiemann (2013), Antonczyk et al. (2009), Bisello (2013), Spitz-Oener (2006)).

David Autor and others is that technological change might affect different workers differently. The canonical model of Skill Biased Technological Change (SBTC) for example features two terms of labour augmenting technology: one augmenting the productivity of high skilled workers and one augmenting the productivity of their low skilled counterparts.[44]

Labour productivity in this model follows a similar structure. In particular we can think of the labour productivity in the model, being summarized by the set of skill weights:

$$\lambda = \left.\underbrace{\begin{bmatrix} \lambda_1^1 & \dots & \lambda_1^N \\ \vdots & \ddots & \vdots \\ \lambda_M^1 & \dots & \lambda_M^N \end{bmatrix}}_{Occupations}\right\} Skills$$

Here the rows summarize different skill groups or occupations and columns represent different work tasks.[45] Since technology only affects labour productivity in this model,[46] the aggregate state of technology is summarized by λ. Rather than being a single aggregate, however, technology in this model is highly heterogeneous, with labour productivity differing across the two dimensions of occupations and tasks.

This provides us with a flexible framework that can not only accommodate Skill or Task biased technological change, but a continuum of hybrid cases, which will be essential for the study of CBTC, since I can selectively affect the productivities of different skills and different occupations and study the resulting distributional consequences.

It should be noted, however that these analyses do not account for changes in the supply of different skills via changes in the skill process. It is therefore only possible to analyze partial equilibrium effects. However, under rigid education markets these effects might provide a reasonable approximation to short run dynamics. In the following exercise I assume that the parameters of the skill process (33a) do not change in response to the changes in technological conditions. Given that we are considering very broad, general skills this assumption might not be very strong. It appears reasonable to assume that the mean or the variance of cognitive ability across the population will not dramatically change over

[44] See Acemoglu & Autor (2011) for an extensive exposition.
[45] Every column represents a set of skill weights $\lambda^n \in \lambda$.
[46] Technological change is Harrod neutral.

the time period (~ 35 years). The same is possibly true for physical health.

To solve the dynamic model, I make use of the technique developed by Boppart et al. (2017)[47], which has become popular for solving heterogeneous agent models with aggregate shocks. Unlike other papers, however I only solve for the deterministic transition path from one equilibrium to the next, since I am interested in investigating permanent technological changes, rather than Business Cycle fluctuations.

9.2 Calibrating the Dynamic Model

The increased usage of computers and related technologies has probably been the most wide reaching transformation of the economy since the second world war. Yet, the effects of the ICT revolution on the labour market are far from unambiguous: computers are said to augment some tasks, whilst substituting for others as well as generating new fields of work activity (c.f. Acemoglu et al. (2014), Gallipoli & Makridis (2018)).

The theory of TBTC holds, that in general technological change affects the demand for different work tasks and the associated skills respectively. This implies, that a change in technology, benefitting a certain skill (for example the IT revolution favouring cognitive skills), should affect wages proportional to how important the relevant skill is in a certain occupation (c.f. Autor et al. (2003), (2008), Acemoglu & Autor (2011)). I make use of the hypothesized relationship between Computer/IT usage and cognitive skill productivity, to calibrate a path for the evolution of the cognitive skill weights:

1. I use 4 waves[48] of the UK Skills and Employment Survey to obtain a measure of how intensely computers are used within a given occupation cluster in a given year. For this I rescale the Likert-Scale answers to the survey question "IMPORTANCE OF: USING A COMPUTER/ PC/ OTHER COMPUTERISED EQUIPMENT" on the interval $[0, 1]$[49], and take the mean across the occupation clusters for each year. In the following I will refer to these values as pc_t^n i.e. the relative value of computer usage in occupation n in year t. The resulting values can be seen in the table below:

The values in the table provide an interesting insight into the way that computerization occurred across different occupations. For Managers, computer usage seems to have steadily increased by about 10% every 5 year interval, while Administrative & Sales workers

[47] See Appendix for a description.

[48] 1997, 2001, 2006, & 2012 - these are the only years that include a question on computer usage.

[49] With 0 referring to essentially no computer usage and 1 corresponding to the case where computers are essential in the performance of ones work duties.

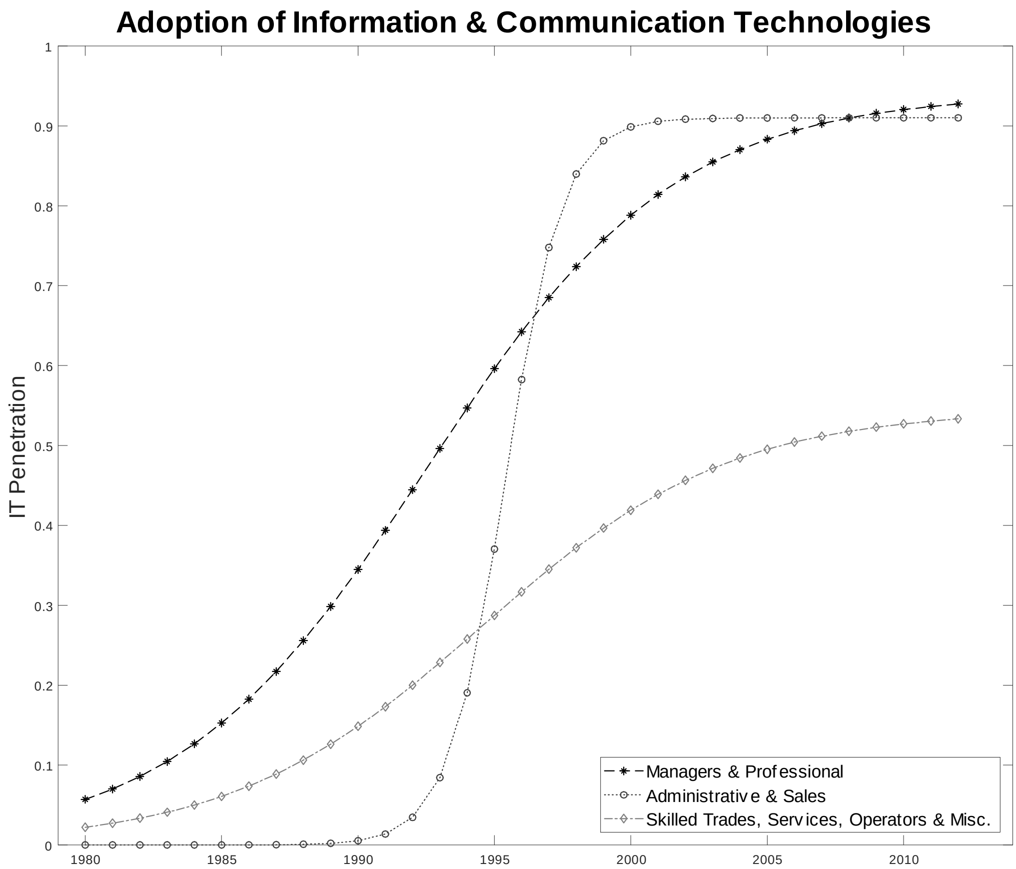
	1997	2001	2006	2012
Managers	0.69	0.81	0.90	0.93
Admin	0.75	0.91	0.89	0.93
Trades	0.38	0.38	0.54	0.52

Table 12: Computer Usage over Time

experienced a big 20% jump around the turn of the millennium. Skilled workers, Machine Operators, Services and Misc. Occupations experience an even bigger jump (roughly 30%) between the 2001 and 2006 waves. I obtain values for the years not covered by the survey, by using interpolation between the available data points, using a logistic functional form, which generates the typical *"S"* shape of technology adoption, that is familiar from work on technology diffusion (c.f. Griliches (1957)).

The following figure depicts the implied adoption curves.[50]

[50] Obviously there is a lot of uncertainty surrounding, specifically, the earlier years of the sample period for which we have no information from the SES. However the general patterns seem reasonable; e.g. the advent of computers in the Administrative cluster coincides with the introduction of Microsoft Office in 1990.


Adoption of Information & Communication Technologies
IT Penetration
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
1980
1985
1990
1995
2000
2005
2010
Managers & Professional
Administrative & Sales
Skilled Trades, Services, Operators & Misc.

2. In order to obtain values for the cognitive skill weights, I assume that computer usage affects the productivity of cognitive skills *log-linearly*, so that a percentage change in computer usage is roughly proportional to a $\varkappa^n$ percentage change in cognitive skill productivity:

$$\ln(\lambda^n_{Cog,t}) \approx \varkappa^n pc^n_t \tag{42}$$

Assuming that $\varkappa^n$ is time invariant, I can eliminate $\varkappa^n$ by dividing both sides of the equation:

$$\frac{\exp^{pc^n_{t+1}}}{\exp^{pc^n_t}} \approx \frac{\lambda^n_{Cog,t+1}}{\lambda^n_{Cog,t}} \tag{43}$$

Hence, for any value of pc^n_t, I can obtain an approximation of $\lambda^n_{Cog,t}$ via:

$$\lambda^n_{Cog,t} \approx \frac{\exp^{pc^n_t}}{\exp^{pc^n_{2012}}} * \lambda^n_{Cog,2012} \tag{44}$$

I use the *5 Quarter Quarterly Labour Force Survey* (5QLFS) to calibrate the year to year occupation cluster transition matrices Π^t. In order to retain comparability, I restrict the sample in the same manner as I have done with the Understanding Society sample. Transitions are measured by movements between the First and Final Interview, in the 5 years centered around the relevant year in order to reduce idiosyncratic measurement error.[51]

I also obtain information about the overall and group specific distribution of incomes, using the value of usual hourly pay and usual hours worked to infer incomes.

9.3 Evaluating Model Performance - Income Inequality

Having calibrated the model and solved for the dynamic transition path, I plot the model responses against those obtained from the data, to evaluate the models performance. Unless stated differently, all data points have been smoothed using a 5 year moving average, to reduce the impact of measurement error.

First we evaluate the models performance with respect to average labour income. In the demeaned data we see a smooth upward trend for the time between 1993 and 2008, and a small decline afterwards, potentially due to the great recession in the wake of the financial crisis. The model captures the upward trend and some of the later slowdown, however does

[51] E.g. the Transition Matrix for the year 2000, is estimated by the transitions between 1998 and 1999, 1999 and 2000, 2000 and 2001 and 2001 and 2002.

not produce as sharp a rise. Still, a regression of the QLFS data on the model generated path reveals a highly significant, positive association, implying that the model is able to explain about 92% of the variation in mean labour income.

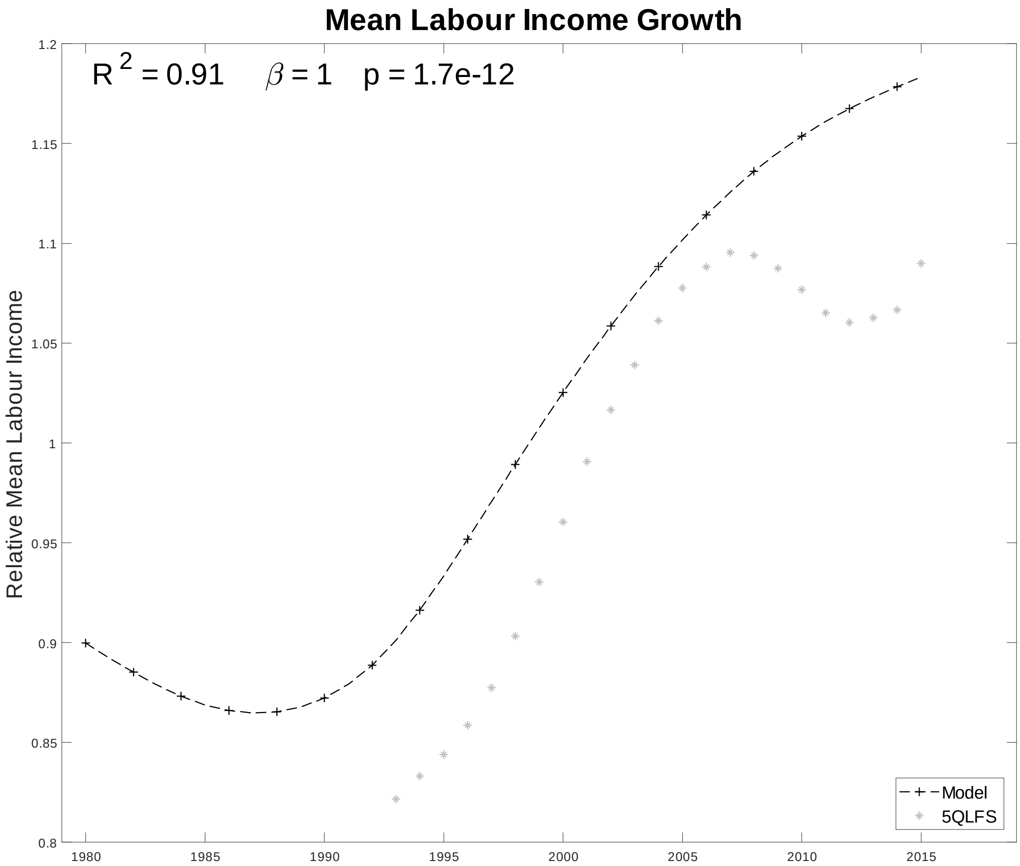


Figure 2: Mean Labour Income

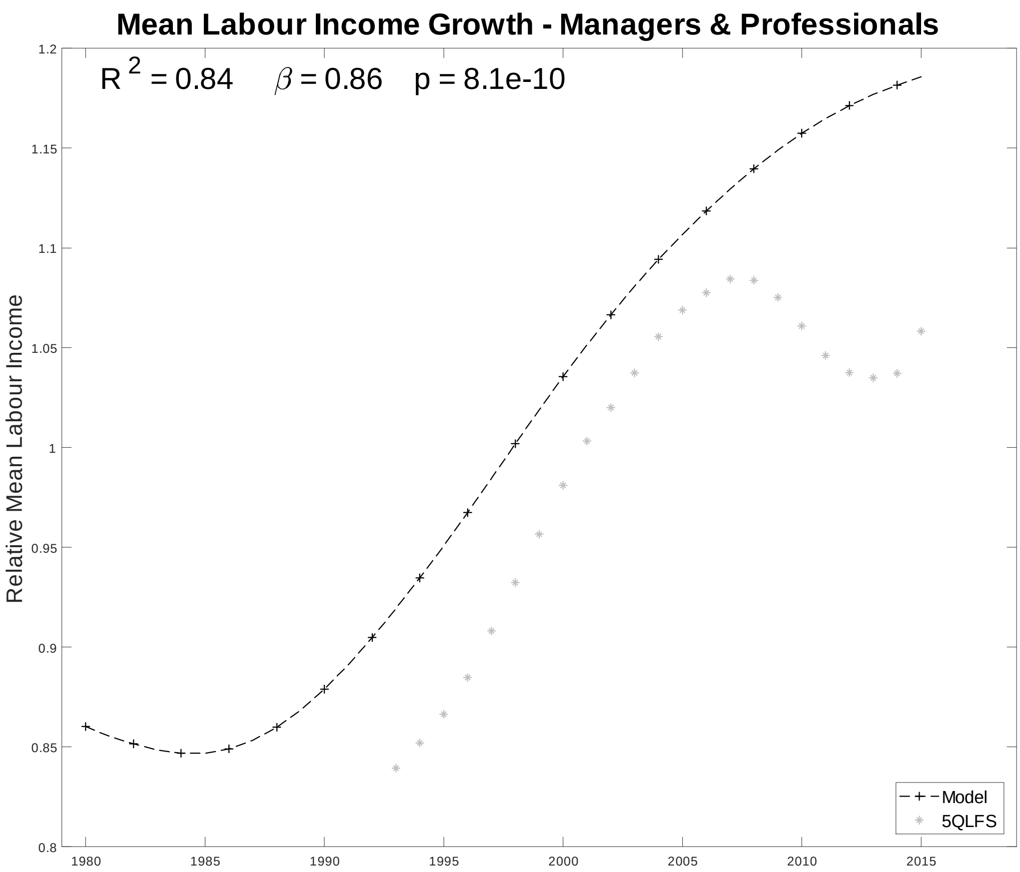


Figure 3: Mean Labour Income - Managers & Professionals

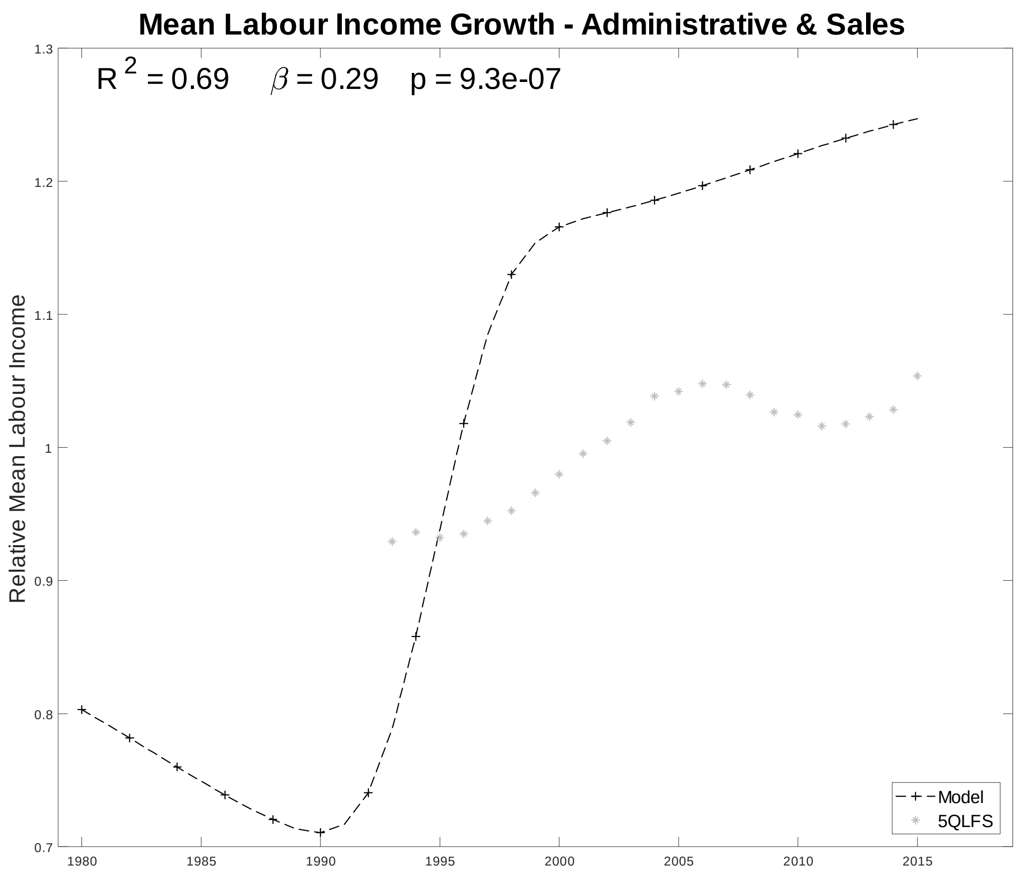


Figure 4: Mean Labour Income - Administrative & Sales

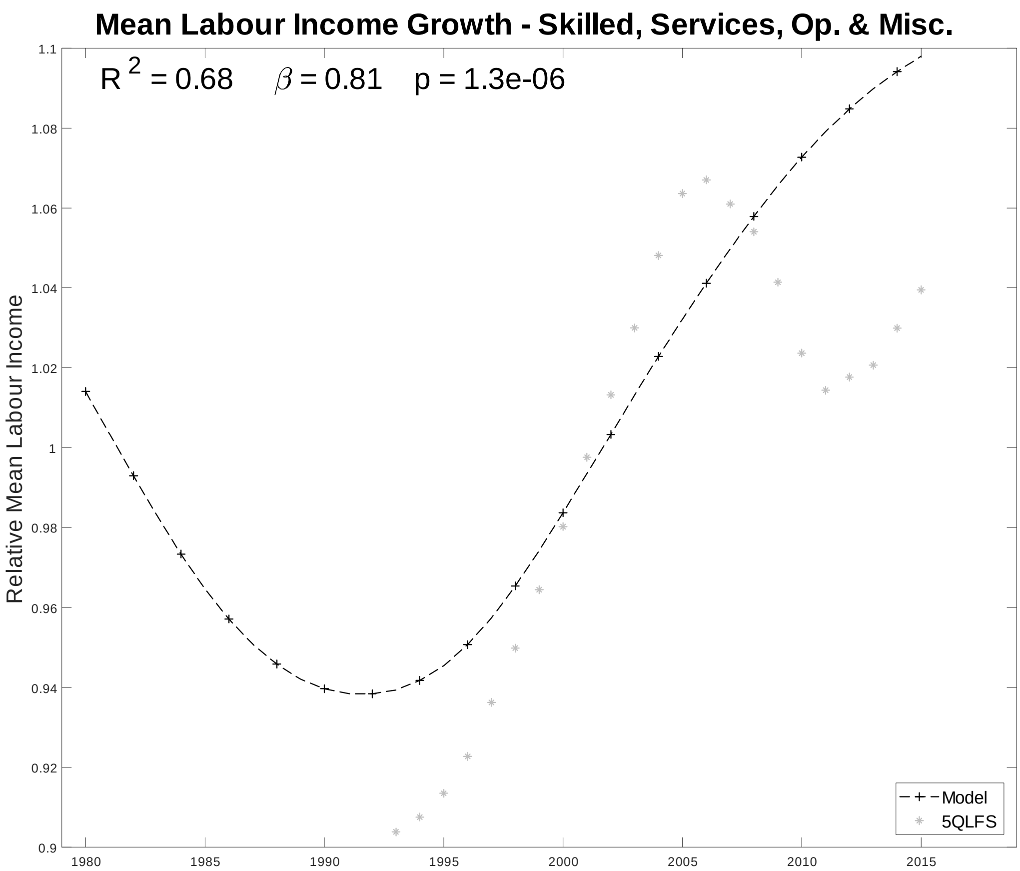


Figure 5: Mean Labour Income - Skilled, Services, Operatives & Misc.

The model also does a convincing job in capturing the mean labour income trajectory of Managers, Administrative and Skilled workers, explaining 84, 69 and 68 percent of the variation in the data respectively.

Further, the model generates a clear "Wage Polarization" effect (c.f. Goos et al. (2009), (2014), Goos & Manning (2007)), with the middle income group of Skilled workers losing their relative position in the income distribution, whilst the lowest paying group (Administrative & Sales workers) catches up over the two decades between 1980 and 2000. Even though the fit with the data is quite weak, the general direction of the model, suggests that a reason for the observed phenomenon of wage polarization could be the comparatively slow computerization of these jobs.

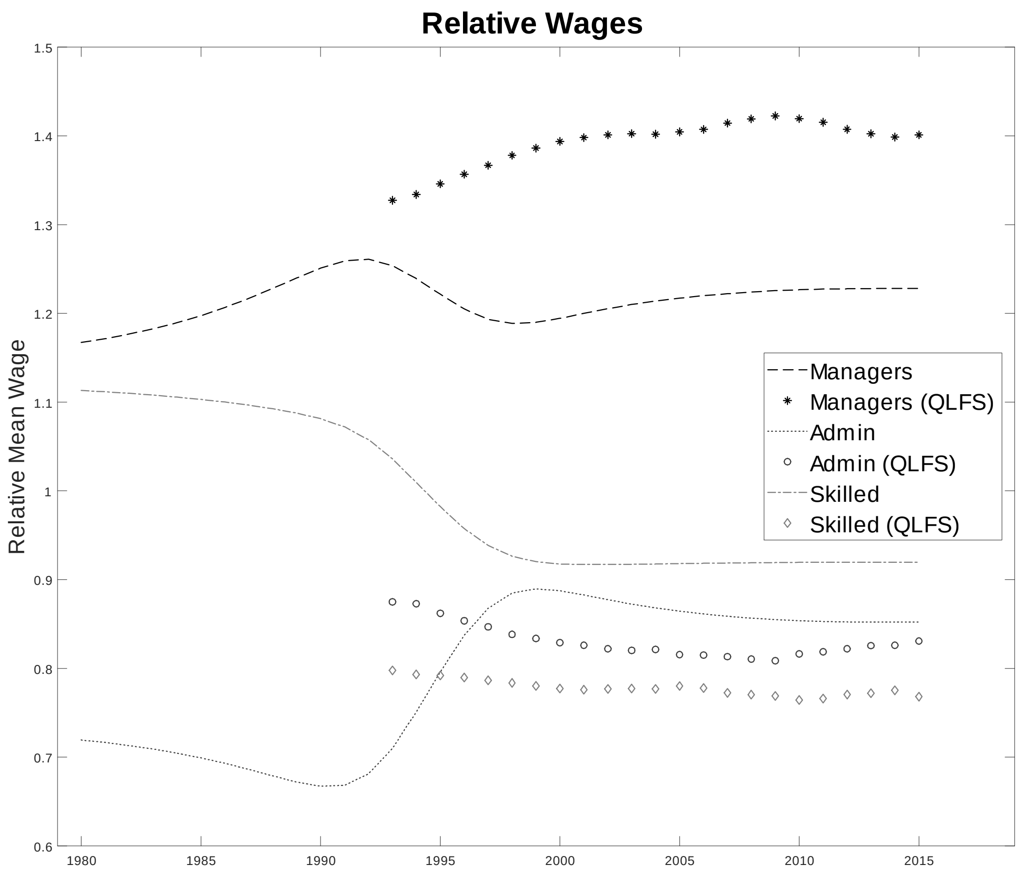


Figure 6: Relative Mean Wages

The model continues to perform well with respect to income inequality as can be seen in the figures below. The fit is even more apparent, when we consider the relative changes to income inequality, by considering the demeaned values rather than the levels. However from both it is apparent that the model manages to account for around 81% of the variation in income inequality as measured by the Gini coefficient. Similar results apply to income inequality as measured by the standard deviation of the logarithms of income. If we consider the possibility, that under sticky wage contracts, changes in labour productivity take some time to show up in measured wages, the fit improves to 88% for a two year lag.

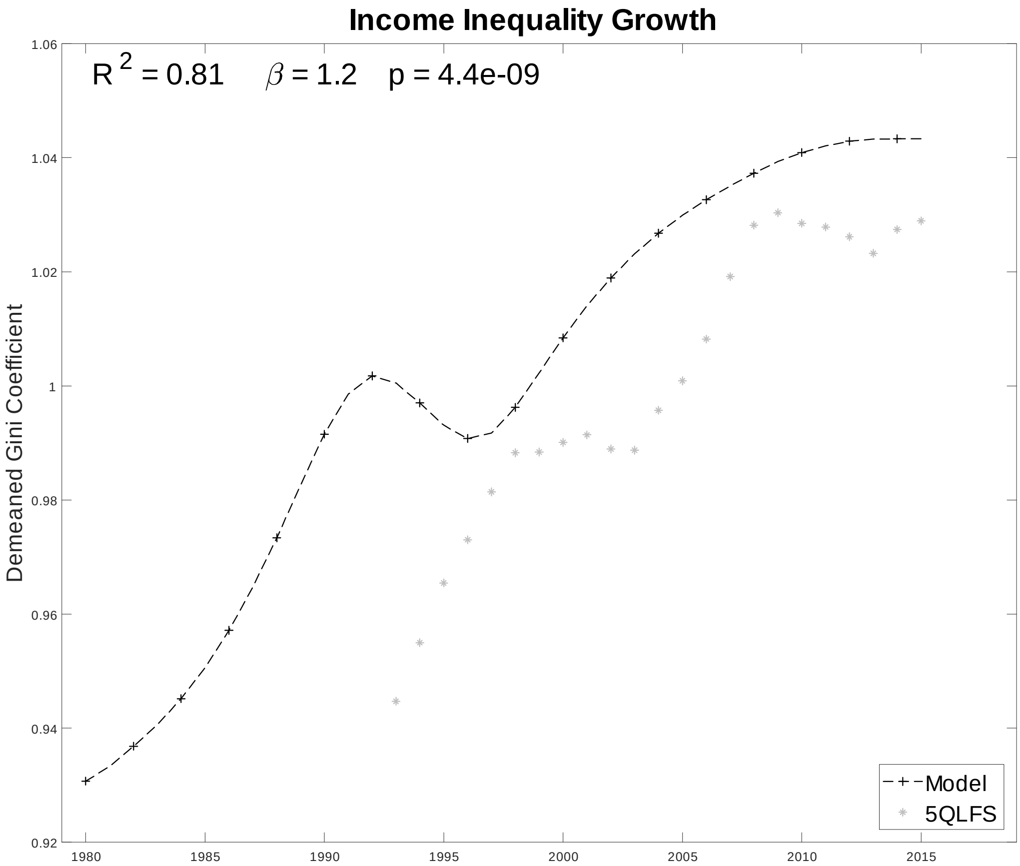


Figure 7: Growth of Labour Income Inequality

Evidently the models fit is not perfect, and there are many important events, such as the 2007/08 Financial Crisis or the Great Recession that will have affected the degree of income inequality whilst being outside of the models scope. Generally, however I hope that this exercise has uncovered some of the general trends due to cognitive skill biased technological change.

9.4 Evaluating Model Performance - Wealth Inequality

There is generally a lot less information about wealth inequality available, especially for such a specific sample of individuals, so confirming the models predictions proves much more difficult.

Hills et al. (2013) provide some Gini values for the UK from $1976 - 2005$ and some additional evidence comes from the Wealth & Assets Survey (WAS), started in 2006. Unfortunately these two series differ considerably in terms of methodology, so it is difficult to obtain a consistent time series that we can evaluate the model against.

So rather than trying to attempt a dynamic comparison as with income inequality, I will outline the general trends that are reported in these two sources, and see if the model matches the stylized facts.

Hills et al. (2013) report some values on adult, marketable wealth, provided by HMRC for the period $1976 - 2005$. In terms of the Gini coefficient the pattern that emerges is the following: wealth inequality is stable around 0.65 for the time period $1976-1995$, whereupon it jumps up to 0.71 in 2000 and remains around that value (0.70) in 2005.

The biannual WAS provides information on aggregate total wealth. As reported by the Office for National Statistics, the WAS puts total wealth inequality at 0.61 Gini points for the years $2006 - 2012$ with an increase to 0.63 in the period $2012 - 2014$ and a slight drop to 0.62 for the two years after that.

Putting these together we can infer two stylized facts about the path of wealth inequality in the UK:

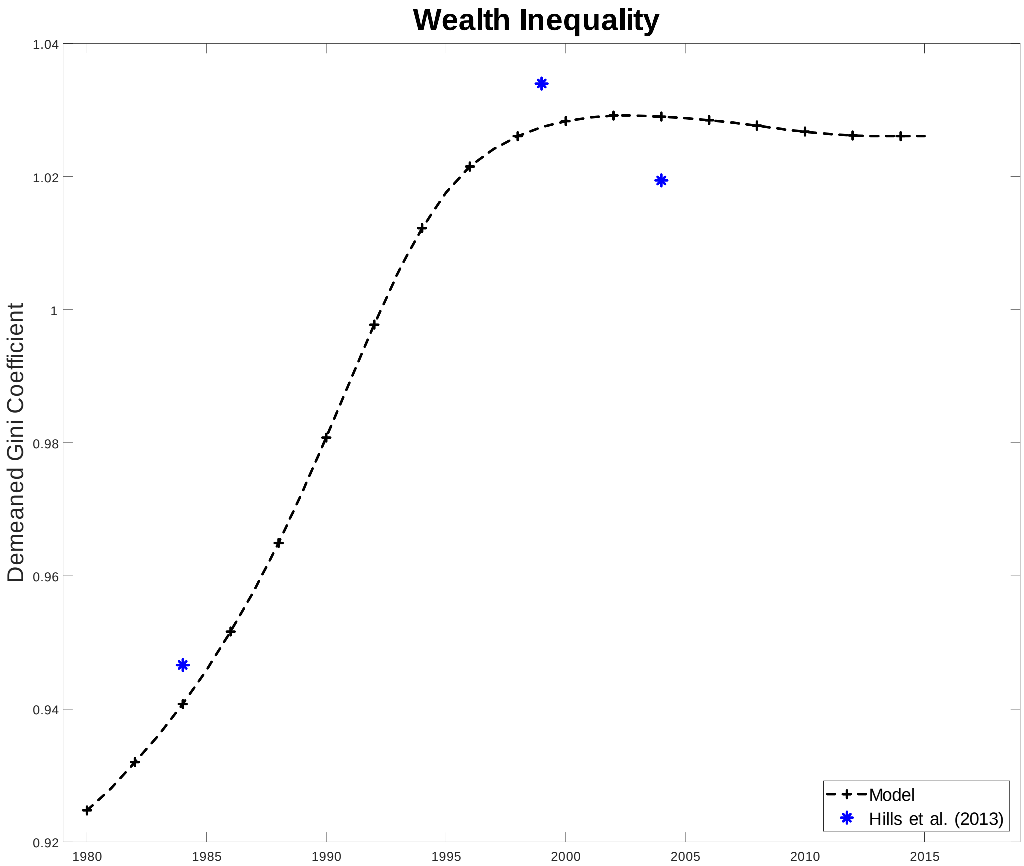
1. A rapid increase in wealth inequality throughout the 1990's, with a peak somewhere around the year 2000.

2. A plateau stage in the first decade of the new millennium, with a (temporary) small bump in the aftermath of the GFC and the Great Recession.

I will use these two stylized facts as criteria to evaluate the models performance.

Again, we should not expect the model to perfectly capture the data. Rather I am interested to see whether the forces exerted by **CBTC** are *qualitatively* consistent with the observables.[52]

[52] Generally there are many forces shaping the distribution of wealth and assets that are difficult to capture with standard models. In particular aspects such as the top 1% share of wealth that has become prominent in the polular imagination of wealth inequality is notoriously difficult to reproduce (c.f. De Nardi (2015)).



Wealth Inequality
Demeaned Gini Coefficient
1.04
1.02
1
0.98
0.96
0.94
0.92
1980
1985
1990
1995
2000
2005
2010
2015
Model
Hills et al. (2013)

As can be seen from the figure above, the model does a decent job at capturing the two stylized facts. There is a substantial increase in wealth inequality, beginning in 1980, peaking in the year 2000, with a leveling off afterwards.[53] Quantitatively the effects are also quite close. Between 1980 and 2000, inequality increased by 9% in the data, which is matched by a 11% increase predicted by the model. In general, the model translates CBTC into a sizable increase in wealth inequality, which is consistent - if not perfectly - then at least with some of the available evidence (c.f. Roine & Waldenstroem (2015)).

10 Policy Experiments

Having shown that the model is able to capture the general effects of technical change on income and wealth inequality, I will use this section to perform some speculative policy experiments. These experiments will showcase some potential technological and policy scenarios, in which the model is used to tease out the (partial) effects on income and wealth inequality.

For these purposes I will set a hypothetical timepath for λ that corresponds to the changes due to technological progress or policy actions, and perform the same analysis as in the previous section, with the difference, that this time the starting point is the present and I am projecting the economy into the future.

It should be noted that - as with any other prediction exercise - the results of these policy experiments are reliant on a huge number of assumptions and are therefore unlikely to be a *correct* representation of future conditions. Rather than those they should be seen as indications of general trends, that will prevail *ceteris paribus*.

10.1 Reversing Automation

One of the phenomena most closely associated with technological change, was the increasing automation of predominantly routine manual occupations, leading to a considerable fall in the incomes of traditional working class dempographics (c.f. Acemoglu & Autor (2011), Autor et al. (2003), Katz & Murphy (1992), Goldin & Katz (2007)). Apart from the purely economic considerations, automation has wide ranging social and political implications: Frey at al. (2018) for example show that in the 2016 US election, districts which were exposed

[53] For obvious reasons the model will not capture the effects of the GFC and Great Recession.

to a higher degree of automation were significantly more likely to vote for Donald Trump. Sooner or later it is likely, that the political establishment will respond to the needs of the losers from technological progress, to avoid a further disaffection of this large share of the electorate.

For this scenario I assume that the government implements a set of policies, aimed at artificially increasing the demand for Skilled Trades, Personal Service, Machine Operators and Misc. occupations, either through trade protections, public employment programmes or industry subsidies (c.f. Atkinson (2015)).

In the framework of the model, this is equivalent to a row shift in the λ matrix: the price of every unit of output by these occupations increases due to the governments intervention, effectively increasing the return to each type of skill in these occupations by a common factor. For this scenario, I assume that the government implements a mix of policies, that immediately raises the mean income of the affected group by 20%, whilst all other factors remain unaffected.

The figures below highlight the response of the endogenous variables to this fiscal policy shock.

The immediate effect is a reversal of *Wage Polarization* as the mean income of the affected workers increases. Correspondingly, the Gini coefficient of labour income falls by approximately 1 point.

Higher incomes lead to increased wealth accumulation from the affected workers, even though within-occupation inequality has not changed. There is some evidence of a small effect on the asset accumulation of Managers and Administrative groups, but the effects are small. Overall the policy reduces wealth inequality by about 0.5 Gini points over the first 10 years and around 1 point after 25 years.

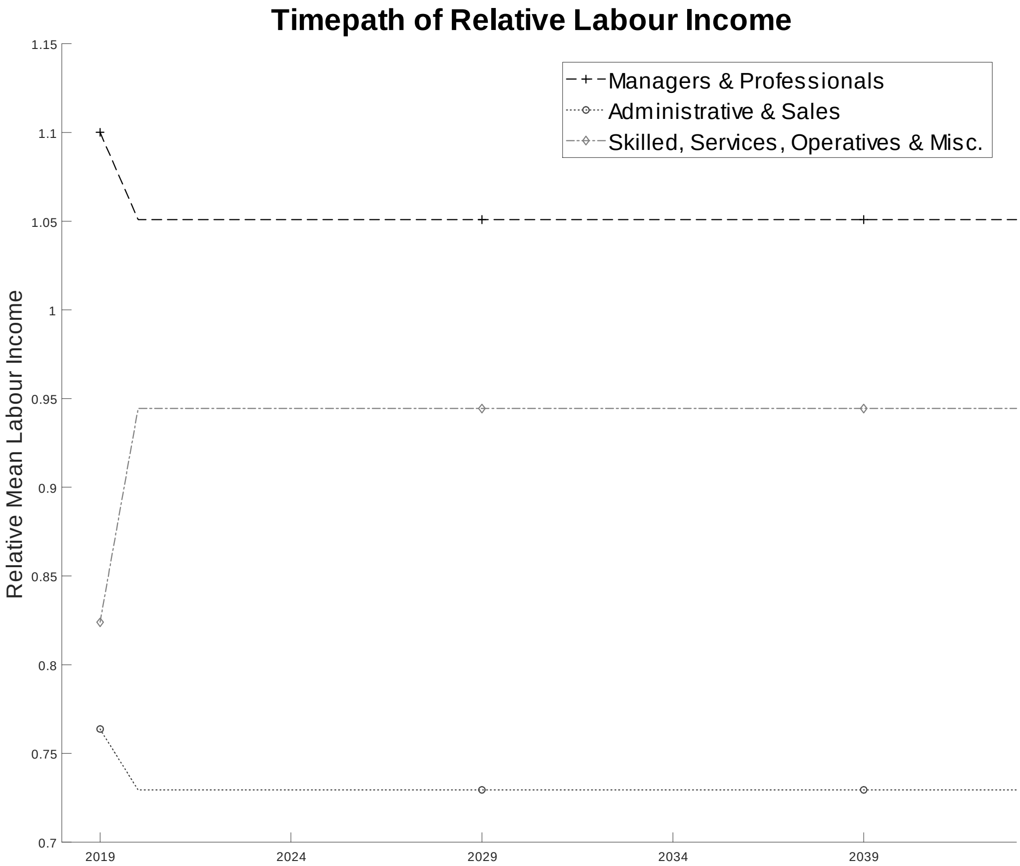


Figure 8: Relative Labour Income - Policy Experiment 1

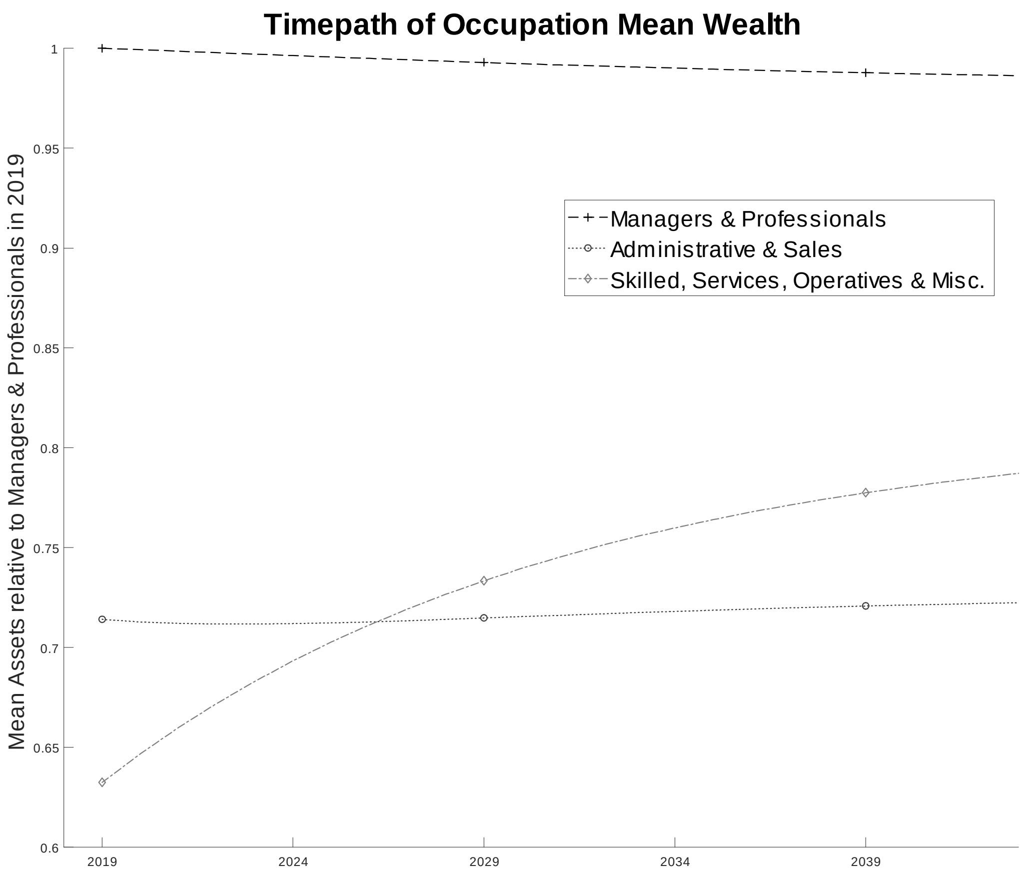


Figure 9: Occupation Mean Wealth - Policy Experiment 1

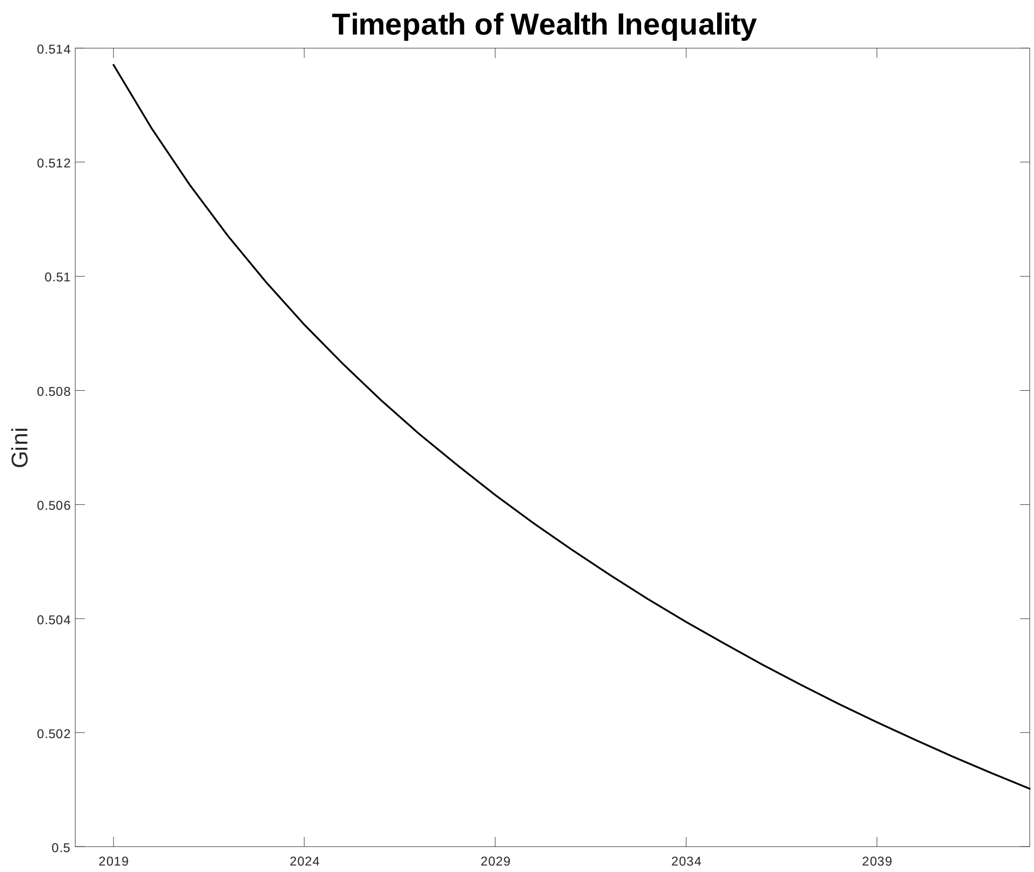


Figure 10: Wealth Inequality - Policy Experiment 1

10.2 Rise of the Machine Brains

With the recent progress in the fields of Machine Learning and Artificial Intelligence, computers are for the first time competing with and outperforming humans in areas such as pattern recognition, logical thinking and strategic decision making (c.f. Adams (2018)). Traditionally these areas have been seen as bastions of the human brains comparative advantage, but as Acemoglu & Restrepo (2017) point out, further progress in these areas might see a fall in the wages of those workers we generally see as the most highly skilled.

In this scenario I simulate the arrival of highly capable artificial intelligence technology, by gradually reducing the cognitive skill weight of Managers & Professionals by 20%.[54] I do not change the return to physical skills, based on the intuition, that the return to those tasks that require a physical, human presence (personal interaction, consultations & negotiations, etc.) will not be adversely affected by these developments.

The figures below show the economy's adjustment to the technological change. The model indicates a gradual compression of relative wages, as incomes at the top of the distribution contract. Correspondingly there is a gradual fall in overall wage inequality on the order of 0.5 (1) Gini points after 5 (10) years. Asset holdings fall gradually, leading to a slight decrease in wealth inequality (0.5 Gini points over 25 years).

[54] I use the following adjustment: $\lambda_{t+1} = (1-\psi)(\lambda_T - \lambda_t) + \lambda_t$, where $\psi = 0.75$ is an adjustment parameter.

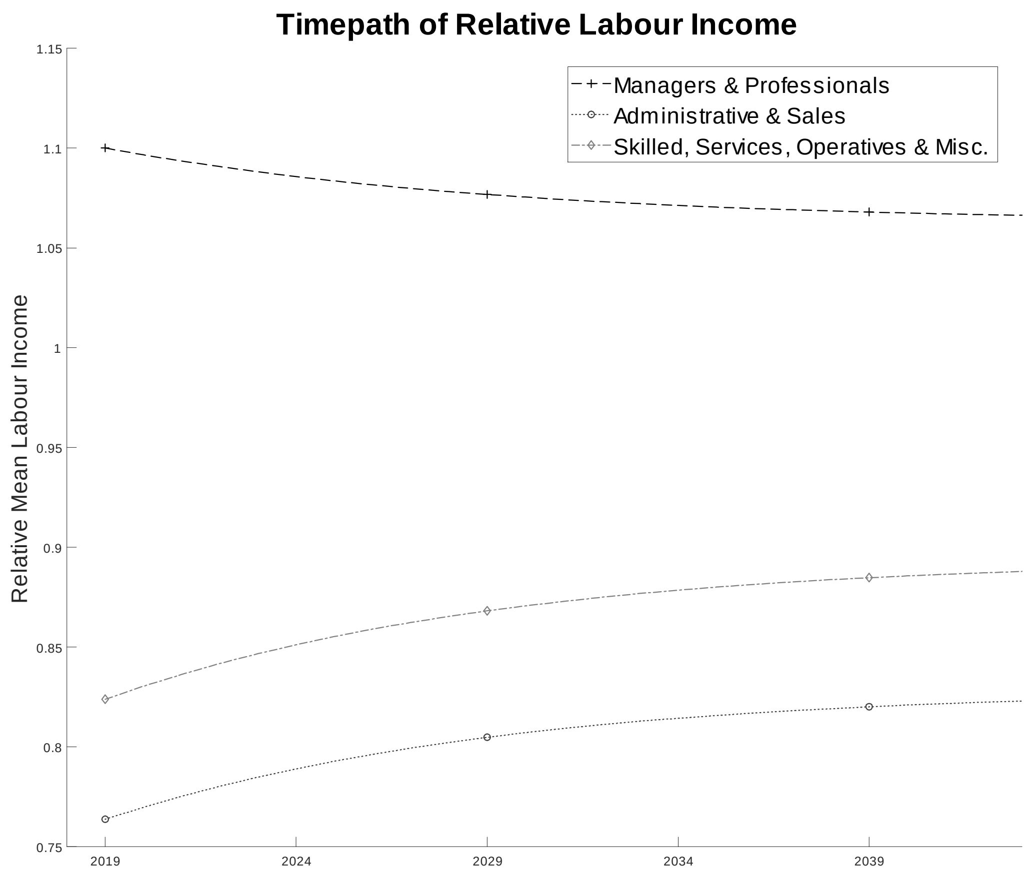


Figure 11: Relative Labour Income - Policy Experiment 2

10.3 The growing importance of "human" skills

The last policy experiment presents itself as the flipside of the previous one. Autor (2015) points out that advances in technology, usually complement those tasks that they don't actively replace. For example, advanced diagnostics technology, would free up a doctors time to consult with patients more thoroughly. Many authors (e.g. Goos (2018)) point out that it is likely that a set of these tasks will be constituted by essentially "human" activities such as social interaction, communication and care. Despite their ability to analyze data rapidly and accurately, a computers social capabilities are still significantly under developed.[55] Furthermore, there might be a strong, qualitative preference for interactions with other humans even if the same service was available from a machine.This trend points towards an increasing demand for the kind of skills that are commonly called *noncognitive* or *interpersonal* (c.f. Edin et al. (2017)).

For this scenario I will raise the physical skill weight[56][57] for all occupations where human interaction is likely to be part of the job.[58]

The results of this experiment are fairly similar to the other two experiments: Income inequality is reduced by around 1 Gini point over the first couple of years, as the technological change takes effect. This is accompanied by a slight increase in wealth for the affected groups.

The response of wealth inequality is somewhat interesting, as it exhibits a hump shape - rising initially, and then falling - however the overall effect is too small to be of much interest.

11 Concluding Remarks

In this chapter I have extended the standard Aiyagari (1994), model with a structural labour income process, that draws on insights from the wider labour literature. Changes in income are not caused by log normal productivity shocks, but rather due to combinations of occupational transitions and changes in an individuals skill-set. I have shown how to

[55] Showcased by the point that no program to date has managed to pass the Turing test convincingly.

[56] It would have been preferable to have an actual measure of social ability such as the Big5 psychometric measures for this analysis. However in order to stay consistent with the rest of the analysis I simply pick physical skills.

[57] Again, the size of the increase is set to 20% with the adjustment parameter $\psi = 0.75$.

[58] These are Managers & Professionals and Administrative & Sales.

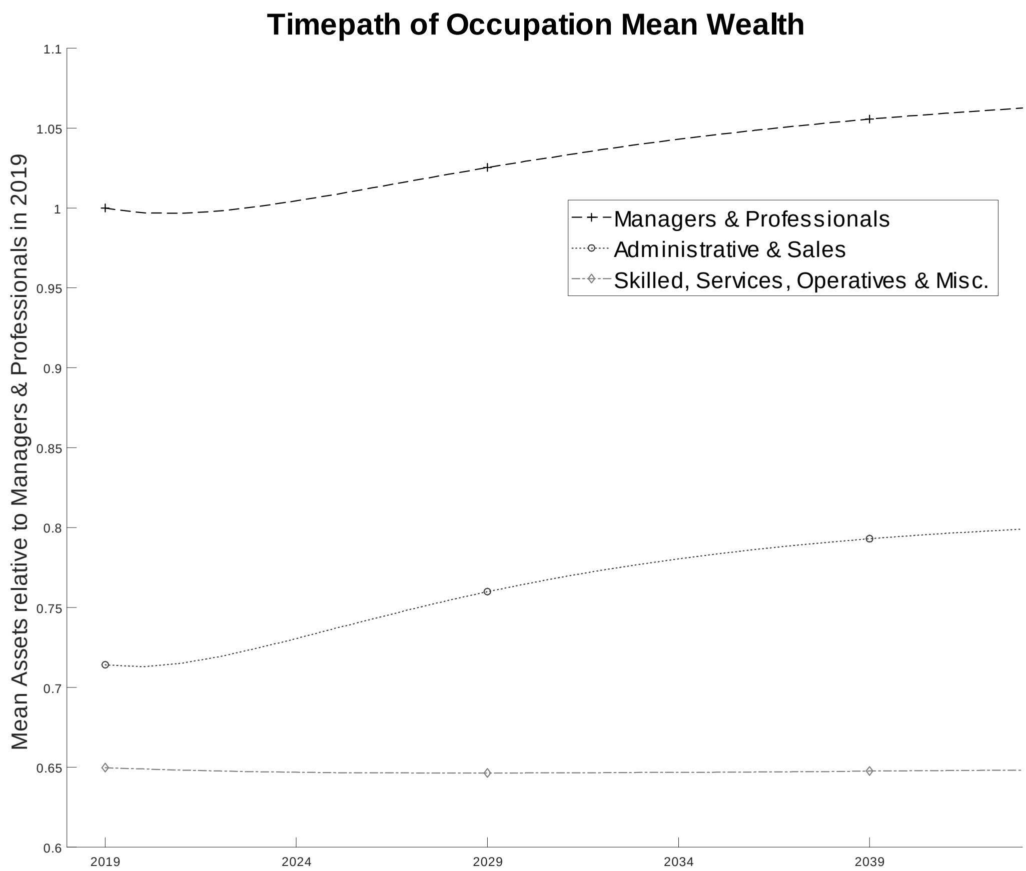


Figure 12: Occupation Mean Wealth - Policy Experiment 3

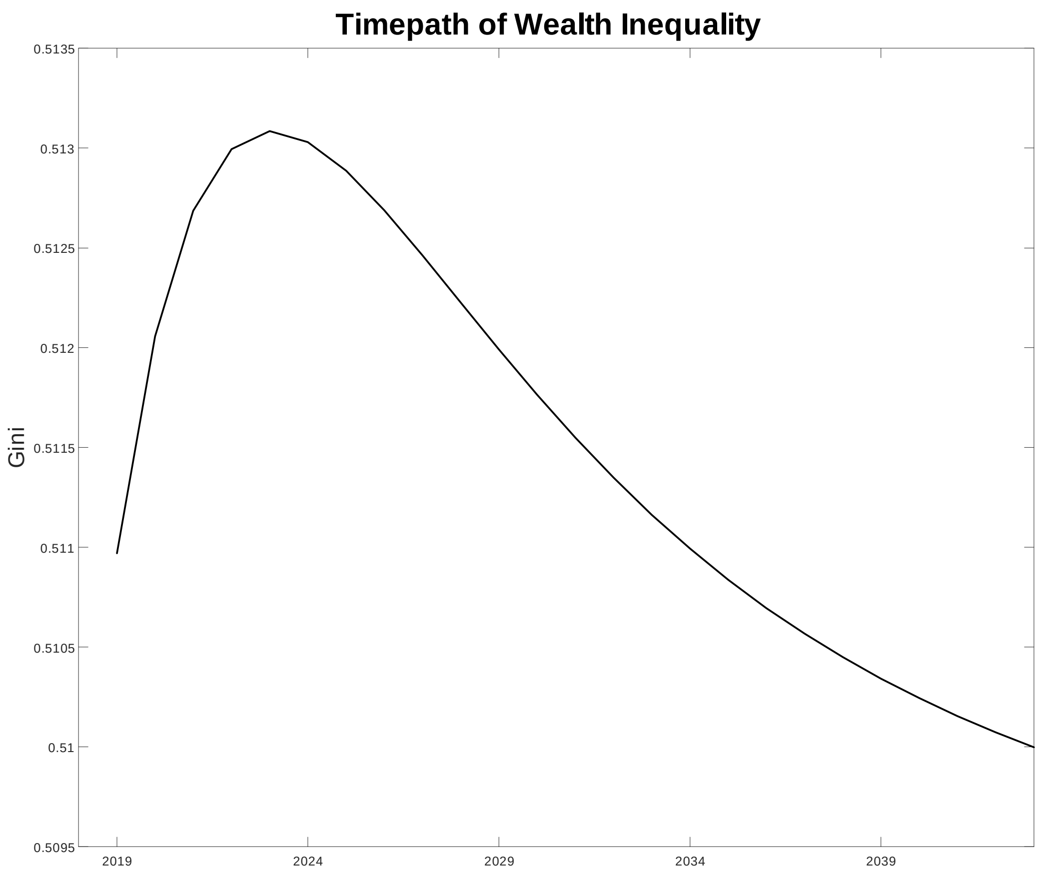


Figure 13: Wealth Inequality - Policy Experiment 3

identify the structural parameters from the data shown that the model can reproduce key moments of the empirical income distribution without actively targeting them.

The model has then been evaluated against available data, using a hypothesized relationship between the importance of computer and IT equipment and the productivity of cognitive skills. The results of the dynamic analysis suggest that the calibrated income process is well suited to capture the developments in average wage income as well as the changes to income inequality resulting from cognitive skill biased technological change.

The analysis suggests that the increased usage of computers and computerized equipment should have contributed to a significant increase in both income and wealth inequality over the period $1980-2016$ in the UK. Although it is not possible here to evaluate the implications for wealth inequality sufficiently, this is consistent with some stylized facts and general analyses of the period (c.f. Roine & Waldenstroem (2015)).

I have also evaluated several counterfactual policy & technological change scenarios, assessing the impact on income & wealth inequality.

Overall this suggests that the model could be useful in evaluating the short to medium run impacts of further technological change on the twin peaks of inequality. With the rising importance of Artificial Intelligence, Automation and the Digital Economy this could be a valuable tool for economists and policy makers alike. For now, however I leave these explorations for future research.

A Appendix

A.1 References

Acemoglu, D. and Autor, D. "Skills, tasks and technologies: Implications for employment and earnings," Handbook of labor economics (4), 2011, pp. 1043–1171.

Acemoglu, D., Autor, D., Dorn, D., Hanson, G. H. and Price, B. "Return of the Solow Paradox? IT, Productivity, and Employment in U.S. Manufacturing", National Bureau of Economic Research, NBER Working Paper, 2014.

Acemoglu, D. and Jensen, M. K. "Robust comparative statics in large dynamic economies," Journal of Political Economy (123:3), 2015, pp. 587–640.

Acikgoez, O. "On the existence and uniqueness of stationary equilibrium in Bewley economies with production," Journal of Economic Theory (173), 2018, pp. 18–55.

Adams, A. "Technology and the labour market: the assessment," Oxford Review of Economic Policy (34:3), 2018, pp. 349–361.

Agrawal, A., Gans, J. and Goldfarb, A. "Artificial Intelligence: The Ambiguous Labor Market Impact of Automating Prediction", National Bureau of Economic Research, 2019.

Aiyagari, S. R. "Uninsured Idiosyncratic Risk and Aggregate Saving," The Quarterly Journal of Economics (109:3), 1994, pp. 659–684.

Altonji, J. G. "Multiple skills, multiple types of education, and the labor market: a research agenda," SSRN Electronic Journal (), 2010.

Angelopoulos, K., Lazarakis, S. and Malley, J. "Wealth inequality and externalities from ex ante skill heterogeneity", 2017.

Atkinson, A. B. Inequality, Harvard University Press, 2015.

Atkinson, A. B. and Bourguignon, F. Handbook of income distribution, Vol. 2, Elsevier, 2014.

Autor, D. "Why Are There Still So Many Jobs? The History and Future of Workplace Automation", Journal of Economic Perspectives 29(3), American Economic Association, 2015, pp. 3-30.

Autor, D. "The task approach to labor markets: an overview", Journal for Labour Market Research 46(3), Springer Nature, 2013, pp. 185-199.

Autor, D. and Salomons, A. "Is Automation Labor Share-Displacing? Productivity Growth, Employment, and the Labor Share," Brookings Papers on Economic Activity

(2018:1), 2018, pp. 1–87.

Autor, D. H. and Handel, M. J. "Putting Tasks to the Test: Human Capital, Job Tasks, and Wages," Journal of Labor Economics (31:S1), 2013, pp. S59–S96.

Autor, D. H., Katz, L. F. and Kearney, M. S. "Trends in US wage inequality: Revising the revisionists," The Review of economics and statistics (90:2), 2008, pp. 300–323.

Autor, D. H., Levy, F. and Murnane, R. J. "The Skill Content of Recent Technological Change: An Empirical Exploration," The Quarterly Journal of Economics (118:4), 2003, pp. 1279–1333.

Baumgarten, D., Geishecker, I. and Görg, H. "Offshoring, tasks, and the skill-wage pattern," European Economic Review (61), 2013, pp. 132–152.

Bayer, C. and Kuhn, M. "Which Ladder to Climb? Wages of Workers by Job, Plant, and Education," (), 2018.

Beaudry, P., Green, D. A. and Sand, B. M. "The great reversal in the demand for skill and cognitive tasks," Journal of Labor Economics (34:S1), 2016, pp. S199–S247.

Becker, G. S. "Investment in Human Capital: A Theoretical Analysis," Journal of Political Economy (70:5, Part 2), 1962, pp. 9–49.

Becker, G. S., Kominers, S. D., Murphy, K. M. and Spenkuch, J. L. "A theory of intergenerational mobility," Journal of Political Economy (126:S1), 2018, pp. S7–S25.

Belfield, C., Blundell, R., Cribb, J., Hood, A. and Joyce, R. "Two decades of income inequality in Britain: the role of wages, household earnings and redistribution," Economica (84:334), 2017, pp. 157–179.

Bisello, M. "Job polarization in Britain from a task-based perspective. Evidence from the UK Skills Surveys," Department of Economics and Management, University of Pisa Discussion Paper (:160), 2013.

Boppart, T., Krusell, P. and Mitman, K. "Exploiting MIT shocks in heterogeneous-agent economies: the impulse response as a numerical derivative," Journal of Economic Dynamics and Control (89), 2018, pp. 68–92.

Carrillo-Tudela, C., Hobijn, B., She, P. and Visschers, L. "The extent and cyclicality of career changes: Evidence for the UK," European Economic Review (84), 2016, pp. 18–41.

Carrillo-Tudela, C., Hobijn, B. and Visschers, L. "Career changes decline during recessions," FRBSF Economic Letters (:2014-09), 2014.

Carroll, C. D. "The method of endogenous gridpoints for solving dynamic stochastic optimization problems," Economics letters (91:3), 2006, pp. 312–320.

Cortes, G. M. "Where have the middle-wage workers gone? a study of polarization using panel data," Journal of Labor Economics (34:1), 2016, pp. 63–105.

De Nardi, M. "Quantitative models of wealth inequality: A survey", Technical report, National Bureau of Economic Research, 2015.

Deming, D. and Kahn, L. B. "Skill Requirements across Firms and Labor Markets: Evidence from Job Postings for Professionals," Journal of Labor Economics (36:S1), 2017, pp. S337–S369.

Deming, D. J. "The growing importance of social skills in the labor market," The Quarterly Journal of Economics (132:4), 2017, pp. 1593–1640.

Fackler, P. and Tastan, H. "A framework for indirect inference," Unpublished manuscript (), 2008.

Firpo, S., Fortin, N. and Lemieux, T. "Occupational tasks and changes in the wage structure," (6), 2011, pp. 28.

Frey, C. B., Berger, T. and Chen, C. "Political machinery: did robots swing the 2016 US presidential election?," Oxford Review of Economic Policy (34:3), 2018, pp. 418–442.

Gallipoli, G. and Makridis, C. A. "Structural transformation and the rise of information technology," Journal of Monetary Economics (97), 2018, pp. 91–110.

Gathmann, C. and Schönberg, U. "How general is human capital? A task-based approach," Journal of Labor Economics (28:1), 2010, pp. 1–49.

Gensowski, M. "Personality, IQ, and lifetime earnings," Labour Economics (51), 2018, pp. 170–183.

Goldin, C. and Katz, L. F. "The race between education and technology: the evolution of US educational wage differentials, 1890 to 2005", Technical report, National Bureau of Economic Research, National Bureau of Economic Research, 2007.

Goos, M. "The impact of technological progress on labour markets: policy challenges," Oxford Review of Economic Policy (34:3), 2018, pp. 362–375.

Goos, M. and Manning, A. "Lousy and lovely jobs: The rising polarization of work in Britain," The review of economics and statistics (89:1), 2007, pp. 118–133.

Goos, M., Manning, A. and Salomons, A. "Job polarization in Europe," American economic review (99:2), 2009, pp. 58–63.

Goos, M., Manning, A. and Salomons, A. "Explaining job polarization: Routine-biased technological change and offshoring," American Economic Review (104:8), 2014, pp. 2509–26.

Griliches, Z. "Hybrid corn: An exploration in the economics of technological change," Econometrica, Journal of the Econometric Society (), 1957, pp. 501–522.

Guvenen, F. "Macroeconomics With Heterogeneity: A Practical Guide", Technical report, National Bureau of Economic Research, National Bureau of Economic Research, 2011.

Guvenen, F., Karahan, F., Ozkan, S. and Song, J. "What do data on millions of US workers reveal about life-cycle earnings risk?", SSRN Electronic Journal , Technical report, National Bureau of Economic Research, Elsevier BV, 2015.

Guvenen, F. and Kuruscu, B. "Understanding the evolution of the US wage distribution: A theoretical analysis," Journal of the European Economic Association (10:3), 2012, pp. 482–517.

Guvenen, F., Kuruscu, B., Tanaka, S. and Wiczer, D. "Multidimensional Skill Mismatch," (), 2017.

Guvenen, F., Ozkan, S. and Song, J. "The nature of countercyclical income risk," Journal of Political Economy (122:3), 2014, pp. 621–660.

Heathcote, J., Storesletten, K. and Violante, G. L. "Quantitative Macroeconomics with Heterogeneous Households," Annu. Rev. Econ. (1:1), 2009, pp. 319–354.

Heathcote, J., Storesletten, K. and Violante, G. L. "The macroeconomic implications of rising wage inequality in the United States," Journal of political economy (118:4), 2010, pp. 681–722.

Hills, J., Bastagli, F., Cowell, F., Glennerster, H. and Karagiannaki, E. Wealth in the UK, Oxford University Press, 2013.

Hubmer, J. "The job ladder and its implications for earnings risk," Review of Economic Dynamics (29), 2018, pp. 172–194.

Huggett, M., Ventura, G. and Yaron, A. "Human capital and earnings distribution dynamics," Journal of Monetary Economics (53:2), 2006, pp. 265–290.

Huggett, M., Ventura, G. and Yaron, A. "Sources of Lifetime Inequality," American Economic Review (101:7), 2011, pp. 2923–54.

Imrohoroğlu, A. "Cost of business cycles with indivisibilities and liquidity constraints," Journal of Political economy (97:6), 1989, pp. 1364–1383.

Kambourov, G. and Manovskii, I. "Occupational Mobility and Wage Inequality," The

Review of Economic Studies (76:2), 2009, pp. 731–759.

Katz, L. F. and Murphy, K. M. "Changes in Relative Wages, 1963-1987: Supply and Demand Factors", 1992, pp. 35-78.

Kopecky, K. A. and Suen, R. M. H. "Finite state Markov-chain approximations to highly persistent processes," Review of Economic Dynamics (13:3), 2010, pp. 701–714.

Krueger, D., Mitman, K. and Perri, F. "Macroeconomics and household heterogeneity"'Handbook of Macroeconomics', Elsevier, 2016, pp. 843–921.

Lazear, E. P. "Firm-Specific Human Capital: A Skill-Weights Approach," Journal of Political Economy (117:5), 2009, pp. 914–940.

Lin, D., Lutter, R. and Ruhm, C. J. "Cognitive performance and labour market outcomes," Labour Economics (51), 2018, pp. 121–135.

Lindenlaub, I. "Sorting Multidimensional Types: Theory and Application," The Review of Economic Studies (), 2017, pp. rdw063.

Lise, J. and Postel-Vinay, F. "Multidimensional skills, sorting, and human capital accumulation", 2016.

Ljungqvist, L. and Sargent, T. J. Recursive Macroeconomic Theory, MIT press, 2012.

Meghir, C. and Pistaferri, L. "Earnings, consumption and life cycle choices"'Handbook of labor economics', Elsevier, 2011, pp. 773–854.

Miao, J. Economic dynamics in discrete time, MIT press, 2014.

Milanovic, B. Global inequality: A new approach for the age of globalization, Harvard University Press, 2016.

Mincer, J. "Investment in human capital and personal income distribution," Journal of political economy (66:4), 1958, pp. 281–302.

Piketty, T. "Capital in the 21st Century," Cambridge: Harvard Uni (), 2014.

Prada, M. F. and Urzúa, S. "One size does not fit all: Multiple dimensions of ability, college attendance, and earnings," Journal of Labor Economics (35:4), 2017, pp. 953–991.

Quadrini, V. and Ríos-Rull, J.-V. "Inequality in macroeconomics", Elsevier, 2015, pp. 1229–1302.

Ríos-Rull, J.-V. "Computation of equilibria in," Computational methods for the study of dynamic economies (), 1999, pp. 238.

Rohrbach-Schmidt, D. and Tiemann, M. "Changes in workplace tasks in Germany—evaluating skill and task measures," Journal for Labour Market Research (46:3), 2013, pp.

215–237.

Roine, J. and Waldenström, D. "Long-run trends in the distribution of income and wealth"'Handbook of income distribution', Elsevier, 2015, pp. 469–592.

Rouwenhorst, G. K. .. Asset pricing implications of equilibrium business cycle models In: Cooley, T.F. (Ed.), Frontiers of Business Cycle Research, Princeton University Press., Princeton University Press, 1995.

Roy, A. D. "Some thoughts on the distribution of earnings," Oxford economic papers (3:2), 1951, pp. 135–146.

Roys, N. and Taber, C. "Skills prices, occupations and changes in the wage structure," (), 2016.

Sanders, C. and Taber, C. "Life-Cycle Wage Growth and Heterogeneous Human Capital," Annu. Rev. Econ. (4:1), 2012, pp. 399–425.

Silos, P. and Smith, E. "Human capital portfolios," Review of Economic Dynamics (18:3), 2015, pp. 635–652.

Slavík, C. and Yazici, H. "Wage risk and the skill premium," (), 2017.

Spitz-Oener, A. "Technical change, job tasks, and rising educational demands: Looking outside the wage structure," Journal of labor economics (24:2), 2006, pp. 235–270.

Stiglitz, J. "The Price of Inequality", 2013, pp. 52-53.

Stinebrickner, R., Stinebrickner, T. R. and Sullivan, P. J. "Job Tasks, Time Allocation, and Wages", Technical report, National Bureau of Economic Research, 2017.

Stokey, N. L. and Lucas, Jr., R. E. Recursive Methods in Economic Dynamics, Vol. 23, Harvard University Press, 1989.

Tinbergen, J. "Substitution of graduate by other labour," Kyklos (27:2), 1974, pp. 217–226.

Tinbergen, J. "Substitution of academically trained by other manpower," Review of World Economics (111:3), 1975, pp. 466–476.

Toda, A. A. "Huggett Economies with Multiple Stationary Equilibria - Google Scholar," SSRN Electronic Journal (), 2017.

Vivarelli, M. "Innovation, employment and skills in advanced and developing countries: A survey of economic literature," Journal of Economic Issues (48:1), 2014, pp. 123–154.

Ware, J. E., Dewey, J. E. and Kosinski, M. How to Score Version 2 of the SF-36 Health Survey:(standard & Acute Forms);[SF-36v2], Vol. 40, QualityMetric, 2001.

Whitley, E., Deary, I. J., Ritchie, S. J., Batty, G. D., Kumari, M. and Benzeval, M. "Variations in cognitive abilities across the life course: Cross-sectional evidence from Understanding Society: The UK Household Longitudinal Study," Intelligence (59), 2016, pp. 39–50.

Yamaguchi, S. "Tasks and Heterogeneous Human Capital," Journal of Labor Economics (30:1), 2012, pp. 1–53.

A.2 Cognitive Skill Measures

Wave 3 of Understanding Society contains an additional module financed by the Economic and Social Research Council (ESRC) with resources from the Large Facilities Capital Fund of the Department for Business, Innovation, and Skills. This module added question stages assessing the cognitive and psychological traits of adult (16+) respondents.

In this chapter I follow Whitley et al. (2016) by selecting 5 exercises from the cognitive module to create a composite measure of cognitive ability. These exercises relate to: 1. Numeric Ability, 2. A Subtraction Exercise, 3. Completion of a Number Sequence, 4. A word recall exercise & 5. Verbal Fluency.

As is apparent from the types of exercises these exercises test the respondents logical, mathematical as well as lingual intelligence. However it is likely that all these areas are related to what is commonly referred to as *general intelligence*. To uncover this latent factor I perform a principal component analysis (PCA) on the standardized test results. The PCA suggest that only the first principal component has an Eigenvalue greater than 1, which is commonly seen as the cutoff for significance. I interpret this insofar that there is only a single factor that is relevant in explaining an individual's performance in all 5 exercises. I dub this factor *general intelligence*, or *cognitive ability.*

The table below reports the factor loadings of the variables. All loadings are positive, suggesting that doing better in any type of test is associated with a higher level of general intelligence, which is what one might expect. The loadings are also of similar size, even though numeric ability and the number sequence exercises have slightly higher loadings.

A.3 Calibration Procedure

In order to calibrate the process for cognitive skills I loosely follow the Method of Simulated Moments (MSM) set out by Guvenen et al. (2014). However, it is probably more accurate

Variable	Loading	Unexplained Variation
Numeric Ability	0.53	0.41
Subtraction Exercise	0.40	0.67
Number Sequence	0.52	0.43
Word Recall	0.39	0.68
Verbal Fluency	0.38	0.69

Table 13: Factor Loadings of PCA on Cognitive Skills

to talk of an Indirect Inference (II) procedure (c.f. Gourieroux et al. (1996)), as the target moments are obtained by running a regression on simulated data. In this case the target moments (m_1, m_2) are the persistence (κ), and the standard deviation of incomes σ_y, observed in the data, as described in equation (41a). The corresponding simulated moments $(\tilde{m}_1(\theta), \tilde{m}_2(\theta))$ are in turn generated by:

1. Picking a vector of parameters θ. In this case $\theta = (\rho_{Cog}, \sigma_{Cog})$, with all other parameters held fixed.
2. Solving the model given those parameters.
3. Simulating a series of labour market histories.
4. Estimating regression (41a), to obtain $(\tilde{m}_1(\theta), \tilde{m}_2(\theta))$.

To implement the procedure, I define $F_n(\theta) = \frac{\tilde{m}_n - m_n}{|m_n|}$, i.e. $F_n(\theta)$ corresponds to the percentage deviation of the simulated n^{th} moment from its target, given the parameter vector θ. I then choose θ to minimize

$$\min_{\theta} F'(\theta) W F(\theta) \tag{45}$$

where W is the identity matrix, as $F(\theta)$ is already scaled. To solve (45) I employ a nonlinear, derivative free, minimization routine.[59]

A.4 Boppart et al. (2017) Algorithm

The solution method follows Boppart et al. (2017):

1. Choose a time T at which the economy has presumably reached a steady state.[60]
2. Solve for the stationary equilibrium at T where the skill weights are given by λ_T.

[59] I also double check, using a genetic algorithm, that the local solution coincides with the global minimum.
[60] For my application I set T= 2200, but most variables settle down after around 20 years.

3. Construct the deterministic transition path of $\{\lambda_t\}_0^T$ and $\{\Pi_t\}_0^T$.
4. Make an initial guess for the transition path of capital $\{K\}_0^T$.
5. Given the guess for the transition path, and the evolution of λ, solve the policy functions backwards from $t = T - 1$, setting $G^T = G^{SS}$.
6. Calculate the transition matrix for the joint state Λ in every time period, using the policy functions obtained in step **5.** Iterate the joint distribution forward, starting with the initial stationary distribution.
7. Calculate the implied capital stock at each point in time and update the guess for the path of capital until convergence.

A.5 Occupation Specific Productivities

In this chapter I sometimes suggested that Managers & Professionals might appear to have high returns to physical skills due to some higher level of unobserved productivity. In the language of the model this corresponds to a third - general - skill, that every worker is evenly endowed with. For simplicity, I set the skill endowment of this general skill to 1. The skill weight, associated with this general skill, can be interpreted as the "base productivity" of the relevant occupation. Ignoring this base productivity, can in theory bias the estimates of the skill weights.

In this appendix section I test this hypothesis, by including Occupation Dummies in the skill weights regression. These dummy variables will capture any higher level of productivity that is unrelated to differences in the skill productivities.

The extended econometric model is given by:

$$\omega_{i,n} = \sum_{n=1}^{N} D_n + \sum_{n=1}^{N} \left\{ D_n \sum_{m=1}^{M} \tilde{\lambda}_m^n s_{i,m} \right\} \tag{46}$$

The regression suggests that excluding occupation specific productivities has indeed affected the skill weight estimates. After including the occupation specific intercepts the estimated returns to all skills have decreased significantly. This is particularly true for Managers & Professionals, who have the highest occupation specific productivity. The general patterns, however are largely unaffected: Managers & Professionals still have the highest returns to cognitive skills, even though they are now very close to the estimate for Admin & Sales occupations. Skilled Workers and other physically demanding occupations still have the highest return to physical skills, even though the return has dropped by around half. Curiously the physical skill weight of Managers & Professionals is still very close to that of Skilled Workers, suggesting that the previous estimates weren't entirely due to higher unobserved productivity.

A.6 Comparison of Skill Weights with SES derived measures

The presented approach to estimating the skill weights, allows for an interesting comparison with the literature. The common approach to recovering skill weights, is based on survey information on individual job content (c.f. Autor & Handel (2013) or Spitz-Oener (2006)).

	Labour Income
Managers & Professionals	934.429***
	(202.988)
Admin & Sales	-530.172
	(319.563)
Skilled Workers, Services, Operatives & Misc	-95.705
	(234.703)
Managers & Professionals X Cognitive Ability	1523.511***
	(188.932)
Admin & Sales X Cognitive Ability	1393.683***
	(268.147)
Skilled Workers, Services, Operatives & Misc X Cognitive Ability	786.205***
	(121.178)
Managers & Professionals X Physical Ability	501.617*
	(220.936)
Admin & Sales X Physical Ability	314.609
	(291.348)
Skilled Workers, Services, Operatives & Misc X Physical Ability	460.493***
	(133.514)
R-squared	0.212
N	4393

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Table 14: Skill Weights Regression with Occupation Specific productivities

For this approach, information on the perceived importance of different tasks is transformed into a measure of the importance of a category of work tasks (such as analytic or manual tasks), and then used as a basis for analysis (see Autor (2013) for an overview of the method and some criticisms).

Since the approach I presented above, differs fundamentally from the commonly employed method it might be useful to cross validate the estimated skill weights with measures derived from job-task data.

For this purpose I employ a wave of the UK Skills & Employment Survey. The UK Skill & Employment Survey (SES) is the UK's primary (and only) source of job task information and contains individual level survey information on different aspects of job content, educational attainment and other demographic factors. I follow Bisello (2013) to select a number of survey items corresponding to analytic and manual tasks, and perform a Principal Component Analysis, obtaining the first principal component in each case. The retained principal components are standardized on $[0, 1]$.

For this I limit the sample to the year 2012, which is closest to the time of the 3rd USoc.

wave (2011 - 2013). I then compare the skill weights obtained from the regression procedure with those implied by the SES data.[61] In an ideal scenario, where workers have a perfect grasp of the contribution that the different work tasks they perform make towards their overall productivity, we should expect a high level of correlation between the skill weights derived from the SES and those estimated from the USoc data.

The correlation between the measures for cognitive and analytic skill weights is 0.66, which warrants some confidence in our measures. The correlation for physical and manual skill weights, however is 0.04 which, at first glance, appears puzzling. A couple of explanations suggest themselves:

1. The SF-12 PCS measure does not correspond well with the tasks covered by the manual skills measure, obtained from the SES.

2. Individuals are unable to properly assess the importance of their manual work activities, or the self reported PCS measures are biased in some unknown way.

3. The correlation might be driven by high estimated physical skill weights for some occupations (e.g. Managers or Professionals) which might in part reflect higher overall productivity in these occupations, rather than a strictly higher importance of physical skills.

To assess this possibility I take the relative importance of skills, by forming the ratio of cognitive/analytical and physical/manual skill weights. The correlation between the transformed variables is 0.35, which still leaves a lot unexplained, suggesting that one of the other explanations is contributing to the mismatch.

Overall this little comparison has provided us with some new and interesting insights: It seems that survey based approaches recover similar skill weights as regression based approaches, at least for cognitive or analytic tasks. This is an important insight since it suggests that at least in this area perceived importance somewhat accurately reflects true labour productivity.

[61] In order to increase the number of observations I use the skill weights obtained for all 9 major SOC2000 occupations.